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Unusual Word Problems and the Development of Critical Thinking in Early School Students

Nietypowe zadania tekstowe a rozwijanie krytycznego myślenia uczniów edukacji wczesnoszkolnej

KEYWORDS

critical thinking, mathematical word problems, unusual problems

ABSTRACT

Nowadays, it is increasingly difficult for people to make their way in a rapidly changing world. Critical thinkers are able to function well in such a changing environment and their education should begin in early childhood.

This paper presents the results of an experimental study on the development of critical thinking of third grade elementary school students. Unusual word problems with missing or contradictory data, an ambiguous solution, or with unrealistic content (meaningless in real life) served as a tool for developing this type of thinking. These problems provoked the students to think, to critically analyze both the content and data of word problems. This, in turn, helped the students become more reflective, notice missing or contradictory data, ambiguity of a solution or lack of realism, fill in missing data, correct contradictory information and make unrealistic data realistic, as well as seek all possibilities for a solution. Thus, their critical thinking skills developed.

SŁOWA KLUCZE

myślenie krytyczne, matematyczne zadania tekstowe, zadania nietypowe

ABSTRAKT

Współcześnie coraz trudniej jest ludziom się odnaleźć w szybko zmieniającej się rzeczywistości. W tak zmiennych warunkach dobrze mogą funkcjonować ludzie myślący krytycznie, a ich kształcenie należy rozpocząć już na etapie wczesnoszkolnym.

W artykule zostały zaprezentowane wyniki eksperymentalnych badań nad rozwijaniem krytycznego myślenia uczniów III klasy szkoły podstawowej. Narzędziem służącym rozwijaniu tego myślenia były nietypowe zadania tekstowe z deficytem lub sprzecznością danych, z niejednoznacznym rozwiązaniem lub o treści nierealistycznej (bezsensowne życiowo). Zadania te prowokowały uczniów do myślenia, do krytycznej analizy treść zadań oraz danych. Dzięki temu uczniowie stali się bardziej refleksyjni, dostrzegali: niedobór lub sprzeczność danych, niejednoznaczność rozwiązania bądź brak realizmu, uzupełniali brakujące dane, korygowali sprzeczne i urealniali nierzeczywiste dane oraz poszukiwali wszelkich możliwości rozwiązania. Tym samym rozwinęła się ich umiejętność krytycznego myślenia.

Introduction

The modern world is changing so rapidly that we sometimes struggle to keep up with these changes. This intensity of changes should be taken into account in children's education, from its very beginning. It is difficult to predict what the world will be like in a decade or so, what skills will be needed in the adult lives of today's early school students. We therefore need to prepare them for life in a dynamic and complex reality, taking into account their needs, abilities and interests. We are supposed to educate people to become independent, capable of both adapting to changes in their environment and introducing necessary and desirable changes in it.

The core curriculum of 14 February 2017 states that "Primary school education is the foundation of education. The school's task is to gently introduce the child to the world of knowledge, to prepare the child to perform the duties of a student, and to implement self-development" (Podstawa programowa 2017: 11). And the purpose of this education is "to develop competences such as creativity, innovation and entrepreneurship; to develop c r i t i c a l [emphasis mine] and logical thinking, reasoning, argumentation and inference skills … to equip students with a body of knowledge and to develop skills that enable them to understand the world in a more mature and structured way" (Podstawa programowa 2017: 11).

The educational goals listed above can be achieved through children's math education. In math lessons, students develop comprehensively, they gather logical and mathematical experience, acquire knowledge and skills specified in the curriculum, practice memorizing skills, precision, perseverance, responsibility, reading comprehension. While solving problems, they learn to analyze facts, synthesize events, estimate risks, make rational decisions, they perfect abstract thinking, and learn to reason and infer correctly not only in familiar situations but also in new, both simple and complex, usual and unusual ones (Podstawa programowa 2017: 26). Learning mathematics develops logical and critical thinking, which is exactly what we need most nowadays and what will be useful to us in the yet unknown future. In problematic situations, we cannot make rational life and work decisions or take a particular stand without it. It helps us in many aspects of our lives. In dealing with a multitude of media information and advertisements, it allows us to make a critical selection and evaluation of their content and identify the relevant pieces thereof. This prevents us from succumbing to manipulation, demagogic tricks of politicians and allows us to make a realistic assessment of their activities. If we do not think logically and critically, we become unreflective. We accept other people's explanations without inquiring into the truth. We make rash and irrational decisions and allow ourselves to be taken advantage of. Therefore, critical thinking is a necessary skill to avoid being manipulated and to be an independent person.

Critical Thinking

Critical thinking[1] is not a clear-cut concept. Various definitions thereof are formulated and interpreted differently depending on the needs. Sometimes it is easier to define what it is not. "Critical thinking is not the same as criticizing, expressing dissent, or the art of argumentation. It is rather an intellectual process through which, by verifying information and combining facts through objective and measurable analysis, we arrive at the truth" (Wichura 2016). Critical thinking is thus the processing of information in a conscious and complex cognitive and metacognitive act.

A person who thinks critically:

- is able to analyze any situation (problem, phenomenon, experience) from many points of view and select from the mass of information the relevant pieces thereof, evaluating it accurately,
- is able to make judgments and assessments based on clear and justified criteria,
- is able to select arguments for and against a thesis and draw conclusions on this basis, predicting their practical consequences,

[1] The words "critical," "criticism" derive from the Greek word *kritikos*. Critical means "based on analysis, examination of the features of an object; analyzing and evaluating a phenomenon, work, etc.; applying the method of scientific criticism" (see entry in dictionary: *Słownik języka polskiego PWN*, 1978: 1065). Therefore, criticism means, among others, the ability to make judgments, see differences and make decisions.

- knows how to arrive at truth using logical inference with true premises,
- seeks to discover and correct both their own and others' weaknesses in judgments, reasoning, and procedures.

Critical thinking is not a skill that arises spontaneously—its acquisition and development involves practice. It therefore requires both effort and training. Exercises in critical thinking can and should be introduced at an early age, for example during math lessons, with word problems as a tool. Word problems are a valuable teaching tool because they are a special case of a problem situation. Therefore, the ability to solve them can also have a practical dimension, being a model (paradigm) of action in any (problematic) situation.

However, not every word problem or the way it is used is a good tool for developing critical thinking. Too many school word problems are convergent in nature, which can and does create the false suggestion that there is always a solution and that there is only one correct solution to a problem situation. In addition, many of these problems, which are intended to illustrate the applications of mathematics, have unrealistic content and show the world in a distorting mirror[2] (Nawolska, Żądło-Treder 2017a; 2017b). When confronted with solving such problems, children start to perceive school as an institution detached from life, and mathematics as a collection of numbers, laws, theorems and formulas without any purpose or use. "Moreover, students cease to notice logical relationships in mathematics and they increasingly often tend to think that it consists of many unrelated arithmetic techniques, each assigned to a specific topic" (Klus-Stańska, Kalinowska 2004: 13). Accustomed to the importance of computations following a predetermined pattern in solving school tasks, they notice the differences between school mathematics and the mathematics they encounter in everyday life. They begin to take math lessons less seriously, perceiving them as a conventional game, a kind of theatre. No wonder, then, that upon entering the classroom, students shut down their common sense, which allows them to function efficiently in the real world, see their own and others' mistakes and judge them properly. They uncritically assume e.g., seven groszy coins [there is no such a coin in the Polish monetary system] or a girl 245 cm tall.[3] In such a situation we can

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[2] The articles Nawolska, Żądło-Treder (2017a, 2017b) contain examples of problems, the content of which is in contradiction with children's knowledge and life experience, which causes difficulties in children's understanding of the world or even shapes a distorted image of it. Such problems make children stop paying attention to the content of the problem, they recognize that it is only a pretext for computations, so they focus on extracting numbers from the text and performing computations. This leads to a lack of problem-solving skills often manifested by inadequate computations.

[3] In 2006, 116 third graders were to solve the following problem: *There are 6 coins in Agatha's piggy bank, a total of 42 groszy. What kind of coins might these be?* As many as 12 students performed the 42 : 6 = 7 division and answered: *These are the seven groszy coins* (Nawolska, Żądło 2007). In 2014, 114 students were to solve the following problem: *Kasia is 10 years old and 145 cm tall. She grew 10 cm over the past year.*

speak of "a peculiar disease of thoughtlessness that the students were infected with, even though it was the last thing that both they and their teachers would wish for" (Klus-Stańska, Kalinowska 2004: 18).

In view of the above, the school's work appears to be a Sisyphean task. It intends to develop in the child what it will need later in adult life in the real world, and at the same time consciously, with great care, does everything to deprive it of contact with this world, keeps it isolated from social life, not giving it the opportunity to experience life's problems (Dewey 1913: 135). "Teachers, being aware … of the difficulties of learning mathematics, try to bring their students closer to the understanding of the concepts, trying to explain everything to them. They also want to protect them from mistakes for fear of fixing them in children's minds" (Kalinowska 2010: 8–9). For fear of these mistakes, they control the student's every step (Kalinowska 2010: 8–9). And yet only active participation in life, solving problems arising from realistic situations can well prepare children for the tasks ahead. Therefore, according to constructivism, in working with children we should abandon teaching and focus on organizing an environment that fosters the activeness of students and thus supports their learning process (Klus-Stańska 2018: 111–130).

In math education, unusual word problems are a good tool for developing students' competence. Solving them contributes to developing the habit of critical thinking, which facilitates the evaluation of various situations, verifying whether there are grounds for adopting a certain thesis, and thus enables the pursuit of truth.

Developing Critical Thinking in Third Grade Students

In the 2018/2019 school year, a pedagogical experiment was conducted in one of Krakow's elementary schools over January and February. The goal was to develop critical thinking in early childhood education students. The study involved only 14 third grade students. The diagnostic tool was a set of 4 unusual word problems. In the preliminary study, the subjects were to solve problems with missing data (1*a*), ambiguous problems—with multiple possible solutions (2*a*), with the content meaningless in real life—unrealistic (3*a*) and with contradictory data (4*a*). Next, an experimental factor—the independent variable—was introduced in the form of specially organized classes aimed at developing critical thinking. In these classes (12 lesson units), the students not only had the opportunity to solve tasks with different types of unusualness, but most importantly they had the opportunity to discuss what they were doing

How tall will she be at age 20? As many as 59 students failed to notice the lack of realism of the situation presented in the problem, made an inadequate mathematization of the content of the problem and gave the result of 245 cm as Kasia's height (Nawolska 2014).

and why they were doing it in the given way. To think of possible and impossible solutions. Whether the information (data) in the sentence is sufficient to solve it, or whether something is missing and if so, how to get the missing data. Whether data gaps can be filled in freely, or whether there are any restrictions, and if so, which ones. How it relates to solving life's problems. They evaluated their ideas, checked their correctness, discussed solutions and wording of the problems, and corrected the incorrect ones.

After a series of experimental classes, a final study was conducted in which the students were again asked to solve 4 unusual problems (1*b*, 2*b*, 3*b*, 4*b*)—different from the preliminary study but with the structure and type of unusualness analogous to the tasks from the preliminary study.

Third grade students' critical thinking skills served as the dependent variable (in both the preliminary and final study), assessed based on:

- the ability to spot missing data and fill them appropriately,
- the ability to see the ambiguity of a solution and to find all possible solutions,
- the ability to recognize a lack of realism and to adjust the content of the task in such a way as to make the task situation meaningful to life,
- the ability to spot contradictory data and to remove such a contradiction.

The students could score 7 points for solving each task in both *a* and *b* versions. Points were awarded for critical and logical thinking skills and this number was not diminished by arithmetic mistakes, lack of written answer (when solved correctly), or shortened presentation (the answer only, from memory). The students could seek a solution in any way they wished as long as it was relevant to the problem situation. Aggregate, score-based summary of the results of preliminary and final study results is presented in Table 1.

Table 1. Score-based summary of preliminary (ps) and final (fs) study results

No.	First name	Number of points for solving the problem								Total		Difference fs-ps
		1*a*	1*b*	2*a*	2*b*	3*a*	3*b*	4*a*	4*b*	Σ ps	Σ fs	
		ps	fs	ps	fs	ps	fs	ps	fs			
1.	Mikołaj	3	7	0	7	0	5	0	7	3	26	23
2.	Sofya	3	7	0	7	0	7	3	7	6	28	22
3.	Zosia	1	7	0	7	0	7	3	7	4	28	24
4.	Mateusz	3	7	0	7	0	7	0	7	3	28	25
5.	Artur	3	7	0	7	0	7	3	7	6	28	22

6.	Przemek	3	7	0	7	0	7	3	7	6	28	22
7.	Alex	3	7	0	7	0	5	0	7	3	26	23
8.	Ania	0	7	0	0	0	5	0	7	0	19	19
9.	Tomek S.	1	5	0	7	0	7	0	7	1	26	25
10.	Tomek St.	3	7	0	7	0	7	3	7	6	28	22
11.	Iza	0	7	0	7	0	5	0	7	0	26	26
12.	Tomek T.	7	7	0	7	0	7	0	7	7	28	21
13.	Maja	1	7	0	7	0	5	0	7	1	26	25
14.	Bartek	0	5	0	0	0	3	0	0	0	8	8
Total		31	94	0	84	0	84	15	91	46	353	307
Arithmetic mean		2.21	6.71	0	6.00	0	6.00	1.07	6.5	2.28	25.21	×

Source: Author's own research.

From the data presented in Table 1, it can be seen that the students did not demonstrate critical thinking skills in the preliminary study (ps). In total, all the students scored only 46 points for solving all the problems in the preliminary study. In the final study (fs), the results were significantly better, as the total score of the entire group was 353. These are just figures. How did the students perform in solving the preliminary and final study tasks? The results obtained are presented in pairs: a problem from the preliminary study and an analogue from the final study.

The Ability to Spot Missing Data and Fill Them Appropriately

In the preliminary study, the students were given the following missing data problem:

Problem 1*a* Piotrek, Paweł and Janek were collecting postcards. Piotrek collected 16 more postcards than Paweł, and Janek collected 14 more postcards than Piotrek. How many postcards did each of them collect?

For the problem to be solvable, we would need to know the number of postcards collected by one of the boys: either Paweł, Janek or Piotr.[4] The problem can also be solved when the total number of postcards of all boys is known.[5]

Only one of the participants (Tomek T.) noticed the missing data: *I did not know how many postcards Paweł had* and filled in this gap by assuming that Paweł had 0 postcards, which made the problem solvable. Thus, filling the data gap, he determined the number of Piotrek's postcards (16) and calculated the number of Janek's postcards (0 + 16 + 14 = 30).

Seven students noticed the missing data and indicated it by writing e.g., *it is not given how many postcards Paweł has*, but they finished their work without filling in the gap. They scored 3 points each.

Three students indicated that it is impossible to solve this problem, but they did not explain why. They scored 1 point each. Another three children wrote that they could not solve the problem because it was too difficult for them. They scored zero points.

In the final study, the students were to solve, analogously to the preliminary study, the following missing data problem:

Problem 1*b* Tomek, Antek and Krzyś enjoy reading adventure books. Tomek read 16 pages more than Antek in a day and Antek read 12 pages more than Krzyś. How many pages each of them read during the day?

As in the case of problem 1*a*, it is not possible to solve problem 1*b* without completing the missing data: the number of pages read by one (any) of the boys or the total number of pages read by all the boys.

All the students noticed that the problem is impossible to solve due to missing data and filled it bringing the problem to a solvable format (to a usual problem). They added one of the numbers: 1, 10, 11, 13, 18, 19, 20 or 30 as the number of pages read by Krzyś. They thus created the easiest variant of the solvable problem. Twelve of them, after filling the missing data, correctly solved the problem scoring the maximum number of 7 points. For example, Ania assumed that Krzyś read 20 pages and calculated the number of pages read by Antek and by Tomek.

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[4] If the number of Paweł's postcards (p) is given, then the number of Piotrek's postcards can be determined as the sum of $p + 16$, and the number of Janek's postcards as the sum of $p + 16 + 14$. If the number of Janek's postcards (j) is given, then the number of Piotrek's postcards can be determined as the difference of $j - 14$, and the number of Paweł's postcards as the difference of $j - 14 - 16$. If the number of Piotrek's postcards (r) is given, then the number of Janek's postcards can be determined as the sum of $r + 14$, and the number of Paweł's postcards as the difference of $r - 16$.

[5] If the total number of postcards of the three boys (s) is given, then the number of Paweł's postcards can be calculated as a third of the difference of $s - 16 - (16 + 14)$. The number of postcards of the other boys is to be calculated as described in footnote number 4.

Two students (Tomek S. and Bartek) mixed up the relationships while solving the problem with the data filled by them and performed only part of the calculations correctly, so they did not score the maximum number of points.

The Ability to See the Ambiguity of a Solution and Explore All Possibilities

In the preliminary study, the students were to solve the following problem with an ambiguous solution:

Problem 2*a* Piotrek is to send a package to his friend. He was told that stamps worth a total of PLN 45 should be affixed to the package. Piotrek has 20-zloty and 5-zloty stamps. How many stamps of each type should he affix?

None of the subjects recognized that there were three possible stamps configurations making up a total value of PLN 45 and that all of them should be included in the answer.

Ten subjects gave exactly one correct way to select stamps. They all included two 20-zloty stamps one 5-zloty stamp.

The remaining students did not give any correct possibility. For example, Bartek determined the sum of all numerical data (45 zł + 20 zł + 5 zł = 70) and at the same time wrote the equation incorrectly (no denomination in the result) and presented the numerical result as the number of stamps.

In the final study, the following problem served as the one with an ambiguous solution:

Problem 2*b* Małgosia loves art activities. She always buys the crayons and paints she needs from the same art supply store. The store's price for the box of crayons is PLN 10, and PLN 15 for the box of paints. During her recent shopping trip Małgosia spent PLN 70 in the store. How many boxes of crayons and how many boxes of paints could Małgosia buy with this amount?

As in problem 2*a*, there are three possibilities of buying crayons and paints for 70 PLN and all of them should be included in the solution.

Twelve students recognized the ambiguity of the solution, listed all the possibilities, and gave the correct answer.

Two students (Ania and Bartek) presented only one possibility. Although Ania initially wrote down three possible solutions to this problem (one of which was incorrect), she backed out by crossing out two of them, including the correct one. In the end, she pointed to buying two boxes of paint and four boxes of crayons as her only solution. Bartek first corrected the data: he changed the price for the box of crayons from 10 PLN to 20 PLN, and the price for a box of paints from 15 PLN to 10 PLN.

Perhaps he liked the very ability to decide on the data, or perhaps his intention was to make the data more realistic according to the boy's experience, or it was an attempt to make computations easier. After changing the data, there were 4 shopping possibilities, but Bartek indicated mathematically only one of them. Bartek and Ania scored zero points for their "solutions."

The Ability to Recognize a Lack of Realism and to Adjust the Content of the Problem in Such a Way as to Make the Problem Situation Meaningful to Life

The following problem served as meaningless in real life in the preliminary study:

Problem 3*a* On the first day Maciek ate 26 chocolate bars and on the second day he ate 5 more than on the first day. How many chocolate bars does he have left to eat if he had 65 of them at the beginning?

Trying to eat as many chocolate bars as outlined in the problem would have to end badly. The content of the problem is unrealistic but none of the students commented on it critically. All the subjects took the problem "seriously" and performed various calculations. Two of them performed correct computations: 65 – 26 = 39, 26 + 5 = 31, 39 – 31 = 8.

Other students were even not able to cope with the arithmetic relationships. Most of them (10 subjects) gave the number 34 in their answer. These students determined the number of chocolate bars eaten on the second day as 26 + 5 = 31 and determined the difference as 65 – 31 = 34 with complete disregard for the number of chocolate bars eaten on the first day.

Ania's work reveals behaviour typical of children who do not analyze problems, but perform computations only. Ania determined the sum of all the figures (65 + 26 + 5 = 96) and gave the answer "Maciek has 96 chocolate bars," that is unrelated to the question.

In the final study, the following problem served as the unrealistic one:

Problem 3*b* A runner has 1.200 km to run. On the first day he ran 480 km, and on the second, 30 km more than the first day. How many kilometers is left for the runner to complete the run?

All the students recognized the lack of realism. They knew that it was impossible to run 480 km in one day, so they revised the problem bringing it to the usual problem format. They changed the unrealistic data value to a more realistic one. Instead of 480, they entered, for example, 15, 17, 20, 48, 50. In addition, they made other changes to the problem, although they were not needed.

Eight of them not only accurately corrected the data, but also correctly solved the problem they created. They scored 7 points each.

As many as five students, when solving the problem using self-corrected data, did not include in their calculations the number of kilometers covered by the runner on the first day. They only calculated the distance covered on the second day and the difference in the length of the entire route and that distance from the second day. They scored 5 points each.

One student, Bartek, made the numerical values so "realistic" that he obtained a contradictory data problem. If he had solved the problem correctly, he would have recognized his mistake. He, however, performed computations inadequate for the content: he determined the sum of all the data and included it in the answer. He scored 3 points for noticing the lack of realism and for correcting the problem.

Ability to Spot and Resolve Data Contradictions

In the preliminary study, the students were to solve the following contradictory data problem:

Problem 4*a* Kasia has 3 ribbons: a pink, a blue and a white one. The pink ribbon is 30 cm long, the blue ribbon is 16 cm shorter than the pink ribbon, and the white ribbon is 17 cm shorter than the blue ribbon. How many cm long is the white ribbon?

The data in the task is contradictory, because with the values given, the white ribbon would have to have a negative length. Data correction could be done in several ways:

- by increasing the length of the pink ribbon,
- by reducing the length difference between the blue and the pink ribbon,
- by reducing the length difference between the white and the blue ribbon,
- by changing one (or both) of the length relationships between ribbons from "shorter" to "longer."

Only five students noted the contradiction in the data. They expressed their observations in different ways. For example, they wrote, that it is impossible to subtract 17 from 14, or they indicated that it is impossible to solve this problem because the white ribbon would be less than 0 cm.

Zosia wrote that "it is impossible to solve this problem because the blue ribbon is shorter than the white one," and Przemek even stated that "Kasia does not have a white ribbon." These students scored 3 points each.

The remaining students (nine subjects) did not notice the data contradiction. In the case of eight of them, this may have been due to arithmetic errors. One of them

(Alex) started the computations, but apparently did not trust his skills, because he did not finish the problem and wrote that "the problem is too difficult for me."

In the final study, the students were to solve the following contradictory data problem:

Problem 4*b* The following flowers were delivered to the florist: tulips, roses and sunflowers. There were 25 tulips, 10 fewer roses than tulips, and 20 fewer sunflowers than roses. How many sunflowers were delivered to the florist?

Thirteen students noted that it is impossible to solve the problem due to contradictory data. Each of them corrected the data bringing the problem to a usual format and correctly solved the corrected problem scoring maximum points. For example, Artur increased the number of tulips to 50 and increased the difference between the number of tulips and roses to 20, although the latter was unnecessary.

One student (Bartek) most likely confused the phrase "10 fewer" with "10 more" and instead of subtracting 10 from the number of tulips (25), he added 10, so he did not notice the contradiction in the data and did not correct it. He scored zero points for his solution.

Conclusion

The data given above show that the students did not demonstrate critical thinking skills in the preliminary study. They did not analyze problem content or data. They were rather unreflective. When solving unusual problems, they tended to select ready arithmetic schemes that were known to them and effective in solving usual problems, although in these cases they were not applicable. They lacked control over their operations and were content with one answer, even though multiple solutions were possible. Does it mean that they are thoughtless? Rather not. This is more a result of the selection of problems in textbooks or the style of the early childhood education teacher. It is sometimes the case at schools that as soon as the content of the problem, a usual one most often, is given, one of the students (the one who functions better in the school system and calculates faster) gives a mathematical formula (operation) as the solution to the problem and the answer, and thus the whole process of solving is finished—those who had too little time to think, understand, try to find a solution, stand no chance, because the teacher gives another task to solve and the situation repeats itself. Again, a few will find the "solution" (operation), and the rest of the students will rewrite it in their notebooks with no time to think about it. In this way, children's potential, their abilities are not being used. Critical thinking is then unavailable to children. In addition, in order to be able to solve problems/tasks, one needs to practice that skill and not every student gets that chance. All it takes is to give children unusual problems to solve and trust them, sow doubts in them and allow them to think, discuss, question and act; the success is then guaranteed. The results of the final study demonstrate that critical thinking is available to early childhood education students. They only need to be allowed to engage in it.

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