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"Book of Nature": How to Teach Children to Read It?

„Księga Natury". Jak uczyć dzieci ją czytać?

KEYWORDS

early mathematical education, "Book of Nature," emotional and volitional competencies, instrumental competencies, measurement

ABSTRACT

Modern science and technology are founded on the belief in the rationality and mathematical structure of the world. Learning about it, the progress and quality of our lives are related to the popularization of thinking about mathematical education as equipping students with the competencies necessary to read the Galilean "Book of Nature." The article presents the idea of a mathematical understanding of reality and the leading emotional-volitional and instrumental competencies that should be provided to students of elementary education in order to shape their beliefs about the effectiveness of this way of cognition and support them in acquiring appropriate knowledge and skills.

In terms of field-specific and social competencies, it is about: awakening cognitive curiosity, building the attitude of epistemic and ethical optimism, belief in the inevitability and cognitive value of error, developing the ability to cooperate and compete in small groups, and to shape the attitude of researcher reliability.

In terms of instrumental competencies, these would be: the ability to model phenomena at the level of substitutes (simulations), knowledge of numbers, decimal positional system and four arithmetic operations, the understanding of measurement and practical knowledge of measures, the ability to problematize phenomena from the natural world, having elementary knowledge of heuristics, a certain level of calculation efficiency and knowledge of basic geometric figures.

SŁOWA KLUCZE

wczesnoszkolna edukacja matematyczna, „Księga Natury”, kompetencje emocjonalno-wolicjonalne, kompetencje narzędziowe, pomiar

ABSTRAKT

Współczesna nauka i technologia ufundowane są na przekonaniu o racjonalności i matematycznej strukturze świata. Jego poznawanie, postęp i jakość naszego życia są związane z upowszechnieniem myślenia o edukacji matematycznej jako wyposażaniu uczniów w kompetencje konieczne do odczytywania Galileuszowej „Księgi Natury”. W artykule przybliżono ideę matematycznego rozumienia rzeczywistości oraz wiodące kompetencje emocjonalno-wolicjonalne i narzędziowe, w które należałoby wyposażyć uczniów edukacji elementarnej, by ukształtować ich przekonania odnośnie do efektywności tej drogi poznania oraz wesprzeć w pozyskaniu stosownej wiedzy i umiejętności.

W zakresie kompetencji kierunkowych i społecznych chodzi o: rozbudzanie ciekawości poznawczej, budowanie postawy optymizmu epistemicznego i etycznego, przekonanie o nieuniknioności i wartości poznawczej błędu, rozwijanie umiejętności współdziałania i rywalizacji w małych grupach oraz o kształtowanie postawy rzetelności badacza.

W zakresie kompetencji narzędziowych byłyby to: umiejętność modelowania zjawisk na poziomie zastępników (symulacji), znajomość liczb, dziesiątkowego systemu pozycyjnego i czterech działań arytmetycznych, rozumienie pomiaru i praktyczna znajomość miar, umiejętność problematyzacji zjawisk ze świata natury, posiadanie elementarnej wiedzy z zakresu heurystyki, pewien poziom sprawności kalkulacyjnej oraz znajomość podstawowych figur geometrycznych.

> Philosophy of nature is written in this enormous book which is continuously open before our eyes, but it cannot be understood unless one first understands the language and recognizes the characters with which it is written. And this book is written in the language of mathematics.
> Galileo Galilei, *Il saggiatore* (after: Van Doren 1991: 200).

Introduction

Mathematical education is an obvious and necessary element of general education, but its place and rank have always depended on the general concept of this education resulting from the general trend of civilization and the arrangement of social life.

For example, ancient Greece produced many outstanding mathematicians who mainly dealt with cognitively interesting, but practically unimportant dilemmas such as: squaring a circle or the so-called "Delian problem" (doubling the cube). Rome did not make a great contribution to the development of mathematics, but using its achievements, created a network of roads and aqueducts, as well as impressive buildings, many of which have survived and are admired by contemporary engineers (Merzbach, Boye 2011).

As it seems, the trend of modern Roman-oriented mathematical education towards the acquisition of competencies by learners in the field of the so-called "applied mathematics" (Hardy 2018: 124), nowadays observed as a good mastery of the formal aspects of mathematics, is far from sufficient without the skillful application of them in various fields of science and industry, which is necessary for the progress of civilization. As Mogens Niss writes, "mathematical competence is the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts. Necessary, but certainly not sufficient, prerequisites for mathematical competence are extensive factual knowledge and technical skills" (Niss 2003: 218).

Such an exposition of "applied mathematics" is not only a civilizational necessity, but also a chance to popularize mathematics itself as a vital competence worthy of attention, useful and manageable in life, unlike the so-called "pure mathematics," the level of which for over two centuries has exceeded the cognitive abilities of the vast majority of people.[1]

The article is devoted to the presentation of the conditions that should be met by effective mathematical education of younger school-age children aimed at developing competencies in the field of "applied mathematics."

What Does Galileo Teach Us?

From the earliest times to the present day, there has been a vivid view that the world is rational and has a mathematical structure, and mathematics itself is not so much a human invention as the result of discovering what God or nature wrote down in the "Book of Nature." Euclid already noted that "the laws of nature are only the mathematical thoughts of God," Johann Kepler wrote simply that "God is a mathematician," and today Godfrey H. Hardy claims that "mathematical reality lies outside us," and "our function is to discover or o b s e r v e it" (Hardy 2018: 123).

The breakthrough in thinking about the world was brought about by the achievements of Galileo and Newton, who proved its mathematical structure, presenting

[1] As Lynn Arthur Steen writes: "Less than 1% of the US population is acquiring elements of mathematics that are beyond the [mathematical knowledge] of the late eighteenth century" (Steen 1983: 15).

equations as the best and surest way to describe and understand reality. In Pierre S. Laplace's mental experiment, a man armed with Newton's mathematical laws of mechanics and augmented but still human capabilities (the so-called "demon"), "could summarize in one equation the motion of the largest bodies in the universe as well as the tiniest atoms; for such a mind nothing would be uncertain, and the future, just like the past, would be before his eyes" (after: Gaarder 1995: 250). Thus, it points to the surprising fact that such equations can also have predictive power, revealing the possibility of empirically confirmed phenomena even after decades.[2]

Assuming that nature is William Paley's "clockmaker" and not Niels Bohr's "gambler," at least as important as obtaining the facts is interpreting them, making them meaningful. They are themselves like a type case with compartments full of types that do not mean anything on their own, and whose meaning is given only by the intention and the thought of those who arrange words and sentences out of them.

"To Want" and "Can." Competencies Necessary to Read the "Book of Nature"

Every action understood as a conscious and purposeful act is anchored in the will.[3] It is also a part of learning about the phenomena of the world, but this will must be equipped with certain intellectual and manipulative tools and the ability to use them effectively. They are complementary goods, and their shape and harmonious combination determine the goals of educational activities that will prepare children to read "The Book of Nature."

Metaphorically speaking, getting to know the night sky requires the possession of a telescope and the ability to use it, and at least elementary knowledge of stellar constellations. Nevertheless, cognitive curiosity combined with the need to spend sleepless nights in silence and solitude is at least equally necessary for this. One needs patience and fortitude to endure inconveniences, failures, adversities of weather and fate.

[2] For example, the existence of a neutrino was empirically confirmed after a quarter of a century and more recently, after half a century, the Higgs boson was discovered.

[3] Action is a specifically human activity "which can be judged morally because of its conscious and purposeful character (*actus humanus*)" (Podsiad 2001: 203).

Emotional and Volitional Competencies

1. Curiosity about the world and persistent inquisitiveness in exploring it

Curiosity about the world is a natural quality of children's minds, but it has at least two perplexing properties. Firstly, they develop a process of habituation to stimuli,[4] a kind of cognitive "anesthesia," which makes them take the phenomena around the world for granted in order to not to be surprised by them or to think about them (Nowak 2010: 12–13; 2016: 71).

Secondly, children's interests are as intense as they are short-lived, and they are not resistant to failure. When asking, children are often satisfied with any answer or, in a rush of amazement, shift their interests to other issues, without waiting for explanations of the previous ones (Ligęza 2006: 235). Such lability and short duration of children's interests are not conducive to acquiring good knowledge and skills, and the teacher should become the children's perseverance, inquisitiveness and will.

I believe that the science learned by children should be more of a story of reaching knowledge, also through mistakes and errors, than just a catalog of their transient successes. More a story of the stubbornness of constant dropping which wears away a stone than of good luck. The story of Maria Skłodowska-Curie's diligence in shifting tons of pitchblende rather than Alexander Fleming's nonchalance and good luck.

2. Epistemic and ethical optimism

We live in the world of a special triumph of science, but also of a crisis of trust in it and world-view revaluations. Significantly, this crisis of trust affects a wide audience as well as scientists, so that even a specific term "ironic science" was coined for this circumstance.[5]

Since students do not live in a social vacuum, they have access to various types of information, whether they like it or not, it is advisable to build students' conviction about the value of scientific cognition and progress made by not avoiding difficult issues.

[4] Jerzy Siuta defines "habituation" as: "an adaptive phenomenon consisting in the reduction of the strength of the reaction and its disappearance when the stimulus causing it continues or when it repeats itself" (Siuta 2010: 98).

[5] "Ironic science resembles literary criticism because it presents points of view, views that are at best interesting and provoking further commentary. It does not, however, lead to the truth. In this way, it is impossible to achieve empirically verifiable surprises, forcing us to make a significant revision of the basic description of reality" (Horgan 1999: 18–19).

The experiences of the last century and the almost daily media reports draw attention to another aspect of the case. In the context of scientific discoveries, more and more questions about the real progress of civilization and the good of changes appear. I believe that we should adopt here the attitude of Maria Skłodowska-Curie, who, when receiving the Nobel Prize, raised this topic saying: "one should wonder whether learning about the secrets of Nature brings benefits to mankind or, on the contrary, this knowledge is harmful to it. … I am one of those who … believe that humanity will learn more good than evil from new discoveries" (Rahoza 2011: 4).

Since human life also has a spiritual dimension, it must be remembered that science does not know and rather will not know the answers to existential questions bothering man also in childhood, and certainly in adolescence. Questions about the purpose and meaning of life, transcendence, good, evil, beauty, etc., fall outside the remit of science.

3. Belief in the inevitability and cognitive value of errors

If "to err is human," then the history of science, which is a catalog of lost ideas, errors and mistakes, is an outstanding example of this maxim. Even ignoring the historical significance of coincidence in the discoveries and the so-called "beatus error," it is worth recalling that the conscious error significantly broadens our knowledge, limiting the scope of ignorance.

Making children aware of this issue is not only a tribute to historical truth, but can also improve the psychological situation of a child who experiments with the world, wanders and becomes easily discouraged, who is situated in a good company of the strongest intellects, which can be comfort in failures and an encouragement to continue cognitive effort.

It is therefore necessary to patiently give students the right to make mistakes and a chance to correct their mistakes, so that they are their mistakes and their "discoveries" as much as possible, despite the fact that, in the face of their redundancy,[6] the teacher, like the divine demiurge, knows the result of experience, and in the face of children's helplessness would like to make use of his competencies.

4. Ability to cooperate and compete in small groups

Research work has long been a team activity. The research is carried out by competing groups of people who inspire and complement each other. In these circumstances,

[6] In the methodology of sciences, in this context, one speaks of subjective and objective research problems, i.e., those to which the answer is unknown only to the researcher (here: a student) and problems to which the answer is unknown to science as such (Cackowski 1964: 105).

one of the important areas of preparation for current and future cognitive activities should be the acquisition of positive experiences related to cooperation and competition by children.

An excellent school of such cooperation, and at the same time—dialectically—of group competition, are various experiences and experiments, which, on the one hand, stimulate more effective effort and, on the other, give a chance to discover synergy.[7] In the course of such activities, students get to know themselves and each other, their strengths and weaknesses, learn to share work and responsibility. They learn to lead others and submit to leadership based on the epistemic authority (subject-related competence) of their classmates and solidarity, indicating that in effective team activities, their coordination and leadership is necessary and beneficial for all.[8]

5. Researcher reliability

Even the brightest ideas and thoughts need to be implemented to become something useful. Researcher reliability, as I understand it, is a competency on the border of field-specific and instrumental dispositions. It requires responsibility, precision in measurements, patience, reliability and regularity in observations.

Instrumental competencies

Effective reading of the "Book of Nature" also requires a number of instrumental competencies; knowledge and skills, which I believe include:

1. The ability to model phenomena at the level of simulation (substitutes)

Mathematical cognition of the world consists in modeling its phenomena in numbers and formulas, but when working with children, it is good to model them using tokens, and then using graphical representations and diagrams. The advantage of such modeling is that it meets the specific thinking of students, but directs their attention towards quantitative phenomena, disregarding the features and physical properties of the modeled phenomena (Treliński 2016: 77–79).

[7] It is about the effect also described as 2 + 2 = 5, where the synchronized joint effort gives a greater effect than the simple sum of the effects of the work of individual team members (Nęcka 1994: 83).

[8] By epistemic authority, I mean authority based on its subjects' belief in the substantive competence of the person who is bestowed with it or who claims it. By deontic authority—one that is related to the performance by the holder of some function, or a person fulfilling some social role. There is always some form of the ability to enforce obedience behind it. Solidarity authority is a deontic authority in which the common goals are pursued both by the one who has authority and those who obey him (Bocheński 1994: 24–26).

2. Knowledge of numbers, decimal positional system and knowledge of the four arithmetic operations and the ability to formalize them

Reading the “Book of Nature” is related to the knowledge of numbers and the actions on them, which are its letters and sentences. From this point of view, it should be noted that the numerical competence of children is much more than knowing, even correctly, a sequence of numerals, and knowledge of actions is more than retrieving verbal and graphical engrams from memory (Gruszczyk-Kolczyńska 2014: 65–70). Having the concept of a number and mathematical operation and a decimal positional system can be compared to a box with a name tag affixed to it, but inside which there is a rich and, above all, diverse and orderly arranged set of experiences, ideas and associations, memory of activities, etc., related to the gradual climbing the steps of enactive, iconic and finally symbolic representations (Dąbrowski 2007: 11). Those who have acquired the concept in such way can use this box by taking out the contents appropriate to the situation, but also create and add new ones to it, which will mean (connotation) and signify (denotation).

3. Understanding measurement and actual knowledge of standard measures as well as relationships and dependencies between them

The places where the created world and the world of numbers meet are the measures. If the numbers are letters, then the measures seem to be the words of the language in which the world is described. The primary issue is therefore the acquisition by students of good concepts, measured, understood and skillfully applied, avoiding degenerate formalism (Hejny 1997: 19). By adhering to the metaphor of “book,” it is about a kind of functional illiteracy,[9] reading measures like words in an unknown language without understanding, which, even if fluent, carries no factual information. This is dangerous because it creates the formal appearance of having appropriate competencies by the students, and thus makes it difficult to recognize the deficiencies and correct them.

It is a situation analogous to the situation when someone reads a thermometer with the Fahrenheit scale (°F) without difficulty, makes appropriate calculations regarding the increase and decrease of temperature, not knowing what this temperature actually means for him, for his well-being.

A realistic understanding of concepts must therefore be built on observations and experiences, on muscle memory, images and imaginations, going through the stages of (I) spontaneous accumulation of knowledge about the physical aspects of the world

[9] Dorota Klus-Stańska, analogically, speaks about the mathematical illiteracy (Klus-Stańska, Nowicka 2005: 107).

by the child in the course of their own activity, (II) intentional, teacher-guided perception and qualitative comparison of items in terms of quantitative characteristics, (III) the child's realization of the necessity of indirect measurement and its essence, (IV) learning the standardized units by the child, (V) performing calculations with the use of nominated numbers, (VI) learning derived units (greater and smaller) than the initial one, their mutual relations and binomial expressions and the ability to convert from measures in larger units to smaller ones, (VII) the creation of a locally ordered system of measures, including measures of extension, surface area, volume and mass as well as the rules for creating derivative units and their names (Nowak 2009: 157–163).

4. The ability to problematize phenomena from the world of nature and the social world

Noticing the phenomena of the world, being curious and surprised by them must be combined with a number of skills that will allow you to problematize this surprise and curiosity. If we assume that the "Book of Nature" is an inexhaustible set of tasks with content, then it would be about the ability to solve tasks and having the necessary competencies:

a) the ability to analyze and assess the initial situation from the point of view of the quantity and quality of information,
b) the ability to generate ideas for a solution and check them, with the ability to move between different levels of representation, and to verify them internally (assessment of the reasonableness and probability of the accuracy of ideas),
c) reflectivity, which makes it possible to evaluate the effectiveness of actions, and above all, in the process of generalizing individual solutions, it leads to their paradigmatization, thus allowing the accumulation of knowledge and skills (Polya 1975: 266–287).

5. Elements of knowledge and skills in the field of heuristic problem-solving techniques

The school, as it seems, rightly aims to equip students with reliable competencies, confirmed as to their effectiveness. However, discovering the secrets of the world, even through play, is a pioneering journey into the unknown and forging one's path, so that those who want to embark on this journey must also be equipped with tools and unusual skills, different from those taken on a school trip. This is the nature of heuristic problem-solving procedures, the knowledge and skillful application of which optimizes the cognitive process, significantly increasing the chances of its success.

There is already a wide selection of literature on the possibility of using heuristics in school, so I would like to highlight only some generalities. Edward Nęcka, an expert on the subject, gives them the form of the following principles:

1. The principle of diversity; there is no one best procedure for reaching knowledge, the more you know, the bigger the chance of success grows.
2. The principle of deferred evaluation, separating in time the usually co-occurring processes of generating ideas and their immediate evaluation, which stifles creativity in favor of common-sense and cautious activities (self-censorship).
3. The principle of rational irrationality; conscious and critical use of intuition, emotions, luck in the cognitive process, etc.
4. Principle of competent incompetence; an intentionally critical approach to recognized epistemic knowledge and authority.
5. The principle of play; the spontaneous and playful nature of the cognitive activities undertaken.
6. The principle of validity; concentration of all cognitive forces on the problem currently being solved (Nęcka 2005:15–18).

As can be seen, the rules are largely paradoxical, which makes them new, surprising and attractive to children.

6. Some level of calculation proficiency and/or skill in using a calculator

It is not a central and intellectually most demanding skill, but it is certainly as necessary as the others. It has the character of an instrumental efficiency that allows for effective (i.e., fast and error-free) testing of ideas and reaching final results.

Without denying that creative anxiety, curiosity and invention are the essence and spring of cognition, let us remember that only turning an idea into numerical data gives information about its truthfulness and possible usefulness.

Under the influence of rapidly developing information technology, calculation efficiency as the goal of education is subject to far-reaching reinterpretation and reduction, but if we treat the process of exploring the world as a creative act, it seems that just like a piece of creation often evolving from its original intention and improving in the course of its implementation, calculation may turn out to be a more creative act than we thought so far.

7. Knowledge of simple geometric figures and their relationship with material reality

Galileo's proposition and the history of mathematics indicate that the beginnings of scientific exploration of the world were geometrical, and that for two millennia

Euclid's *Elements* were an imitated but unattainable example of scientific accuracy and clarity.

The world of civilized man is a geometric world, it is a space of circles, triangles and rectangles, cubes and spheres. It is delineated according to parallel and perpendicular lines, it is built according to levels and verticals, and as a result it is impossible to understand the world without knowing even the elements of geometry.

Apart from that, as Urszula and Gustaw Treliński point out, for the student geometric education may additionally be:

1. the way to the ability to mathematize the relations of the surrounding world,
2. a "bridge" between the natural language and the language of mathematics,
3. a factor conducive to the development of mathematical imagination and intuition,
4. an important element of shaping an active attitude towards tasks and problems (Treliński, Trelińska 1996: 7–8).

In order to avoid degenerate formalism, also in this area, one should strive to discover the properties of figures and objects having their shape, and not focus on unnecessary, incomprehensible, and sometimes simply misleading definitions (Gruszczyk-Kolczyńska 2009: 371–376).

Conclusion

Reading and understanding the "Book of Nature," like any other book, requires appropriate development of both core and instrumental competencies. Developed enough for the students to be able not only to apply them in new situations, but using a kind of "generative grammar" to create such situations themselves, ask questions about them and formulate hypotheses to solve them.

Among the field-specific and social competences, attention should be paid in particular to the importance of:

1. curiosity and cognitive inquisitiveness,
2. optimism about your own cognitive abilities and the good that comes from it,
3. conceptions about the inevitability of making mistakes and their cognitive value,
4. skills of effective cooperation within a small group and competition with others,
5. research reliability and precision of measurements.

Instrumental competences, necessary in rational cognition of the world, should include:

1. the ability to model phenomena at the simulation level,
2. knowledge of natural numbers, understanding the decimal system of counting and arithmetic operations,
3. understanding the essence of measurement and practical knowledge of standard measures,
4. the ability to problematize phenomena from the world of nature and social life,
5. knowledge and practical ability to use simple heuristic techniques,
6. good calculation efficiency at the level appropriate for a given class,
7. knowledge of simple geometric figures and the ability to interpret world phenomena in their categories.

Shaping the cognitive attitudes of children taking into account these elements may seem to be a chance for more effective understanding, because they are based on practical activities, in learning mathematics through them. Building their sense of self-agency in this regard, and in the future, they will undertake more frequent studies and work in disciplines where the knowledge of "applied mathematics" is one of the basic competences. These are disciplines that determine the further development of our civilization and the quality of our life.

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