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Developing Guided-Discovery Activities to Support Mathematical Learning: Concepts by Preschool and Early School Pedagogy Students

Konstruowanie zajęć ukierunkowanych na odkrywanie wiedzy matematycznej: koncepcje studentów pedagogiki przedszkolnej i wczesnoszkolnej

KEYWORDS

early school education, mathematical guided-discovery learning, didactic concepts of students

ABSTRACT

The search for teaching methods that facilitate more effective learning of mathematics is one of the challenges faced by contemporary education. Although effectiveness is semantically closer to economic notions, the term can also be applied when assessing the school learning process. In defining what we believe to be the tell-tale signs of effective learning of mathematics, our attention has been drawn to developing skills that facilitate a broad application of knowledge specific to this discipline both within and outside its scope. Guided discovery is a method that enables pupils to construct mathematical knowledge while improving their understanding of it. This study aimed to discover the concepts preschool and early school education students have for constructing guided-discovery lessons to support mathematical learning, as well as their opinions on the attractiveness and usefulness of the suggested solutions. Quantitative and qualitative analyses of student-proposed lesson plans and questionnaire results indicate difficulties in designing lessons using this method, while recognizing their educational value.

SŁOWA KLUCZE

edukacja wczesnoszkolna, nauczanie ukierunkowane na odkrywanie wiedzy matematycznej, koncepcje dydaktyczne studentów

ABSTRAKT

Poszukiwanie metod nauczania matematyki umożliwiających poprawę efektywności kształcenia tego przedmiotu jest jednym z wyzwań współczesnej edukacji. Efektywność, mimo że w swej semantyce bliższa jest kategoriom ekonomicznym, to jednak może odnosić się do oceny szkolnego procesu kształcenia. To właśnie określenie tego, co według nas świadczy o efektywności nauczania matematyki, kieruje naszą uwagę w stronę kształtowania umiejętności pozwalających na szerokie zastosowanie osiągnięć tej dziedziny w jej zakresie, ale także poza nią. Jedną z metod, która umożliwia uczniom konstruowanie wiedzy matematycznej przy jednoczesnym pogłębianiu rozumienia, jest kierowane odkrywanie. Niniejsze badanie miało na celu poznanie koncepcji studentów pedagogiki przedszkolnej i wczesnoszkolnej w zakresie konstruowania lekcji metodą nauczania ukierunkowanego na odkrywanie wiedzy matematycznej, a także ich opinii na temat atrakcyjności i użyteczności proponowanego rozwiązania. Analiza ilościowa i jakościowa scenariuszy studenckich i wyników ankiet wskazuje na trudności w zakresie projektowania lekcji tą metodą przy jednoczesnym dostrzeganiu jej walorów edukacyjnych.

Introduction

Children encounter mathematics long before they start school, experiencing it through various everyday situations. This includes not only handling small numbers and identifying shapes or selecting objects but also describing observed patterns. Due to the ubiquitous presence of mathematics in our culture, these activities come naturally to children, who do not see them as formal learning. Unfortunately, in educational practice, what initially feels natural to children often becomes difficult and disconnected from everyday life through school teaching, leading to challenges in learning mathematics (Gruszczyk-Kolczyńska, 2012). This highlights the need for effective teaching strategies that address the diverse educational needs of students, showcase the usefulness of mathematics, identify learning difficulties early, and provide timely interventions. We are looking for teaching approaches that will motivate learners to undertake intellectual challenges and help them consciously and actively build their mathematical knowledge.

Learning mathematics through discovery

Mathematics education should heavily rely on those methods that enable students to construct knowledge independently, develop analytical and metacognitive processes, unleash creativity, improve problem-solving skills, and recognize the extensive applications of mathematics in everyday life. Thus, the educational practice of mathematics, seen as the search for and description of regularities, focuses on creating teaching situations that facilitate students' discoveries. It is the discovery of laws, rules, relationships and dependencies that has been recognized as a valuable way to develop mathematical thinking and reasoning (Zazkis and Liljedahl, 2002; English, 2004; Clements and Sarama, 2009). It is often stressed in the literature that children should explore, discover, compare, and describe mathematical regularities from an early age (Clements, 2001; Frobisher and Threlfall, 2005; Garrick et al., 2005; McGarvey, 2012; Kalinowska, 2017). These opportunities are provided by learning through discovery, which is part of heuristic methods (Bereźnicki, 2004, p. 318).

The subject of learning through discovery has been extensively studied, and empirical research indicates its positive effects on cognitive, emotional-volitional, and psychomotor development (Kamaluddin and Widjajanti, 2019). One of the most significant advantages identified in enhancing students' mathematical competence is the ability to pursue knowledge independently, analyze multiple solutions to problems, and cultivate learning skills, resulting in students' intellectual empowerment (Liljedahl, 2005). Undoubtedly, learning mathematics through discovery fosters readiness to initiate and modify different ways of doing things (Dixon, 2005; Kapur and Toh, 2013; Mason et al., 2005). Other benefits include better attitudes of learners toward mathematics and toward themselves as problem-solvers (Svinicki, 1998), and improved appeal of school classes, leading to higher student engagement in the classroom (Carroll and Beman, 2015). The active involvement associated with guided-discovery learning enhances student self-efficacy and teaches the importance of organizing information, planning solutions, and verifying results.

Developing the ability to transform and verify information helps students consider multiple solution paths, which, according to Lockhart (2009), supports critical thinking. The strong emotions that accompany discovery leave a lasting memory imprint and facilitate access to stored information (Svinicki, 1998; Westwood, 2008), while frequent reorganization of this information ensures it remains relevant. Teaching through discovery creates opportunities for students to generate and expand ideas and to correct their results. This enhances student self-reliance in developing more efficient problem-solving strategies (Boaler, 2016; Gopnik et al., 2004; Lu, Bridges, Hmelo-Silver, 2014). Well-constructed tasks become learning challenges that emphasize mathematical skills such as noticing regularities, patterns, and connections; comparing; organizing and reorganizing information; as well as making inferences and formulating hypotheses (Maarif, 2016; Yurniwati & Hanum, 2017).

One important aspect of using this method is the opportunity for students to discuss their observations with peers, which fosters their mathematical literacy. By incorporating more mathematical concepts into their statements and describing the operations they perform, students deepen their understanding of mathematics (Maarif, 2016). Additionally, discussing math problems helps students develop the ability to evaluate their actions, compare results, and seek alternative solutions. This process cultivates a reflective approach to mathematics, where argument-based discussions encourage students to check their strategies and results against the task's requirements.

An undeniable advantage of this method is the increase in academic achievement (Moore, 2005; Amiyani and Widjajanti, 2018), better comprehension of mathematical concepts (Maarif, 2016), development of higher-order cognitive thinking (Yuliani and Saragih, 2015; Amiyani and Widjajanti, 2018), and enhanced self-regulation skills (Fauzi and Widjajanti, 2018). In conclusion, learning through discovery has been found to be more effective than traditional knowledge transmission methods and is recognized for its value in improving students' mathematical abilities (Sahara et al., 2018).

Leading learning through discovery

Although the essence of learning through discovery is that learners construct their own meanings, leaving them unsupported in this process would be a mistake (Mayer, 2004; Lazonder and Harmsen, 2016; Abrahamson and Kapur, 2018). In school practice, this teaching method focuses on the discovery of mathematical knowledge. It falls under problem-based teaching methods and, like them, it posits guiding students through solving practical and theoretical issues by means of inquiry and investigation (Okon, 2001). The goal is to challenge students, inspiring them to consider objects in new ways, described by previously unnoticed relationships and revealing previously overlooked properties. This necessitates viewing mathematical knowledge in new terms and contexts. Children organize and present these insights in new forms, leading to a deeper understanding of mathematics (Van Joolingen, 1999) and empowering them to overcome learning difficulties (Kirschner et al., 2006). In this approach, new concepts, rules, or properties are the culmination of students' intellectual efforts, providing the tools to tackle more tasks and mathematical challenges.

Let us assume, therefore, that teaching aimed at the discovery of mathematical knowledge involves organizing activities to create conditions for students to build their own understanding from data and information obtained through their observations and experiments (Hanum, 2018). At the start of the lesson, the teacher presents a mathematical challenge designed to provoke and engage students, encourage them to ask questions, make and test hypotheses, and seek solutions through analytical and intuitive heuristic methods. The lesson structure, like the problem-solving process, follows specific stages (Galant, 1987, p. 94). First, the teacher engages the students and focuses their attention on exploration. Next, the learners collectively gather information on what they already know about the topic. Then, in groups, the students work on solving the task. Throughout the problem-solving process and during the presentation of their results, they share their findings and collectively sum up new insights. The final stage of the lesson focuses on understanding where and how they can apply their newly discovered knowledge.

This lesson structure mirrors the stages of the research process and can be treated as such in the context of mathematics education. An essential element of this approach is the activation of the communication space, where conversations about planned and completed activities and the exchange of insights stimulate reflection. This helps students better understand the issue at hand and view it from different cognitive perspectives. It is precisely through social exchange, negotiation, and cooperation that individual knowledge is constructed (De Corte, 2013, p. 89).

The most important structural element of such a lesson is a task whose solution is supposed to lead students to mathematical knowledge that is new to them. Guided-discovery tasks differ from investigative tasks and problem-solving tasks. Guided-discovery tasks have a well-defined goal and the content contains the necessary clues to guide students to achieve it. Here, unlike in investigative tasks, the end result is more predictable (Yeo, 2017). It is for this reason that Jaworski (1994) recognizes that exploring is fundamentally different from the research process, and compares it to marking a trail to a place planned in advance by the teacher.

The most important structural element of such a lesson is the math problem, whose solution is intended to lead students to new mathematical knowledge. Guided-discovery tasks differ from investigative and problem-solving tasks. Discovery tasks have a well-defined goal, and their content includes necessary clues to guide students toward achieving that goal. Unlike investigative tasks, the outcome of discovery tasks is more predictable (Yeo, 2017). Jaworski (1994) notes that exploring is fundamentally different from the research process, and compares it to marking a trail to a destination planned in advance by the teacher.

Methodology

I introduced the method of discovery-based learning to the students during a lecture. Following this, I invited them to participate in a lesson where they took on the role of students and discovered the triangle inequality. After the demonstration lesson, a discussion ensued, during which students could ask questions to clarify any doubts they had. However, understanding does not necessarily translate to the ability to design a lesson. Hence, I decided to conduct a study.

The main aim of the study was to understand the perspectives of students of preschool and early childhood pedagogy regarding the construction of mathematics lessons using the discussed method and to thoughtfully analyze them. I aimed to reconstruct students' perceptions of how such a lesson should be conducted, including which mathematical topics should be selected and what tasks should be used to guide learners in discovering mathematical concepts. Additionally, I sought to gather students' opinions on such lesson organization. This aspect is crucial in a teacher's work, as individual attitudes toward the proposed teaching method influence its frequency of use in educational practice.

I crafted the following research questions:

1. What concepts do students of preschool and early childhood education utilize in designing mathematics lessons through discovery-based teaching?
2. What mathematical content do students deem suitable for implementation via discovery-based teaching?
3. Which stages of students' engagement during a lesson taught using discovery-based teaching do they incorporate into their scenarios?
4. How do they formulate tasks aimed at uncovering mathematical knowledge?
5. What are students' perspectives on the effectiveness of discovery-based teaching in learning mathematical concepts?

The study comprised two phases. Initially, students created math lesson plans aimed at exploring mathematical concepts in early childhood education. In the subsequent phase, students completed a brief questionnaire gauging their opinions on the effectiveness and appeal of the discovery-based teaching method, and reflected on their own schooling experiences. Both quantitative and qualitative methodologies were employed to analyze the gathered data. Quantitative research utilized a 5-point Likert scale (Babbie, 2003), with survey questionnaire reliability assessed using Statistica 13.1. When scrutinizing students' lesson plans, the focus was on qualitative analysis, particularly content evaluation (Pilch, Bauman, 2011, pp. 350–354). Several aspects were delineated, including students' selection of mathematical content suitable for this method, stages of student involvement considered in lesson planning, and the construction of tasks pivotal to student exploration. Moreover, the lesson stages in student submissions underwent further quantitative analysis to ascertain their frequency in lesson scenarios.

The study enlisted 184 preschool and early childhood education students and was conducted from March to May in 2021 and 2022.

Analysis and findings

Students' preferences in topics of instruction

Leaving it up to the students to choose the class topics was not a random decision. I wanted to understand the breadth of mathematical content where students see applications for the discovery-based teaching method. Analysis of students' work revealed that the method was primarily applied to mathematical laws and rules. Specifically, there were numerous instances of the commutative law of addition (53 scripts) or multiplication (45 scripts), the associative law of multiplication (18 scripts) or addition (22 scripts), and the separability of multiplication from addition (19 scripts). Fewer projects proposed teaching mathematical concepts such as polygon perimeter (5 scenarios) and units of weight and length (9 scenarios). Some touched on properties of even or odd numbers (7 scenarios) or geometric figures (3 scenarios). Moreover, students suggested topics like "From the fact that something has a larger size, it does not follow that it is heavier" (1 script) or "The larger the unit of measurement we adopt to describe a certain size of a given object, the smaller the numerical result we will get" (2 scenarios). The students' choices indicate a focus on topics explicitly stated in the general education core curriculum. However, propaedeutic content, which prepares students for concepts in higher grades and fosters mental development, was notably absent from their projects.

Students' preferences in planned student activities

The students' lesson plans were designed to outline the activities planned for the students, with the teacher's role being to support them in making discoveries. Among the main tasks of the teacher in this approach, we can identify providing students with tasks that allow them to perceive and form new knowledge, engaging students in the lesson and maintaining their cognitive curiosity, also through inspiring them to ask questions and make generalizations (Dylak, 2000, pp. 71–72). The culmination should be demonstrating the practical application of the students' newfound knowledge.

The findings from the analysis of the student scenarios are shown in the table below, which outlines six key stages of the lesson (Pawlusińska, 2021, pp. 100–107) and describes the approaches proposed by the students. The last column of the table indicates the percentage of scenarios in which the given solution was identified.

Table 1 Student concepts included in lesson plans

Lesson Stage	Student suggestions included in lesson plans	Percentage of occurrence
CREATING A SPACE OF STUDENT EXPLORATION	Introducing the topic of the lesson through factual questions (What is it? Where have you encountered it? Do any of you know what it means?)	19%
	Introducing the topic of the lesson through procedural questions (How should it be calculated? How should it be performed?)	14%
	Mathematical puzzles, games and activities covering the mathematical content previously learned by the students	6%
	"A maths warm-up," such as a series of short problems to calculate	24%
	Omitted from the scenario	37%
ENGAGING STUDENTS	Introducing the purpose of the lesson using hypothetical questions (What's going to happen next? What's the next element?).	32%
	Introducing the purpose of the lesson using conjectural questions (Why is something happening? What is it for?)	23%
	Introducing the purpose of the lesson with speculative questions (What do you think will happen if...? What will happen if...?).	3%
	Omitted from the scenario	42%
TASK TYPES	Guided-discovery tasks	19%
	Procedural tasks	81%
PRESENTATION OF THE RESULTS	Presenting the results and student discussion of the results	62%
	Student presentation of the results, followed by teacher evaluation	38%

Lesson Stage	**Student suggestions included in lesson plans**	**Percentage of occurrence**
ARTICULATION OF NEW KNOWLEDGE	Articulation of new concepts (rule, property, definition...) by students	57%
	Provision of new knowledge (rule, property, definition...) by the teacher	32%
	Omitted from the scenario	11%
SEEKING APPLICATIONS FOR THE NEW DISCOVERY	Solving tasks from the textbook and practice materials	24%
	Students creating their own examples of tasks utilizing their discoveries	15%
	Students seeking applications of discovered knowledge in everyday life	9%
	Extending students' explorations, i.e., what happens when we change the conditions in the task (e.g., , does the discovered property still hold for three and more numbers?).	4%
	Omitted from the scenario	48%

Gathering information about students' past experiences serves two important purposes. Firstly, it helps collect information about their current knowledge and its sources. Secondly, it establishes a foundation for inquiry to tackle new problems. That is why I paid close attention to how students planned to utilize existing knowledge and engage students in discovering new knowledge. An analysis of the student lesson plans revealed that not all plans included these stages of the lesson. Surprisingly, 37% of scenarios skipped the stage of gathering information about students' experiences. Among those that did include it, there were questions covering basic features of mathematical objects (factual questions in 19% of the lesson plans) and inquiries about calculations or determining specific quantities (procedural questions in 14% of the lesson plans). Additionally, in 30% of scenarios, so-called "maths warm-ups" were planned. These consisted of mental math exercises, including single-operation math problems provided by the teacher (24% of scenarios), and short didactic games or puzzles (6% of scenarios), such as:

Topic: Commutative property of addition

> Riddle: Kasia and Tom gathered chestnuts and acorns in the park. Upon returning home, they discovered Kasia had gathered 9 chestnuts and Tom had gathered 8 acorns. Before putting them in the box, they decided to count their combined collection. Kasia combined Tom's acorns with her chestnuts and counted 16. Tom combined Kasia's chestnuts with his acorns and counted 18. How is this possible? (Student lesson activity).

The planned activities for pupils were also driven by questions. The predominant types of questions used were hypothetical (32%), conjectural (23%), and speculative (3%). Although their importance in extending student exploration was highlighted by Wittmann in the second half of the last century (Krygowska, 1977), there were very few speculative questions. Unfortunately, four out of ten student lesson plans did not include this stage.

Analyzing the questions proposed by the students in the context of "thinking through questions," we see that their suggestions included questions from all three levels: gathering information (33% of the scenarios), organizing information (23% of the scenarios), and creating and expanding information (35% of the scenarios) (Szmidt, 2006). Information-gathering questions were planned for the initial stage, when students aimed to collect information about their pupils past experiences. In contrast, questions focused on organizing, creating, and expanding information were planned for the parts of the lesson designed to engage students in exploration.

An important aspect of analyzing the scenarios was determining how students construct tasks intended to lead to the discovery of new knowledge. To do this, I examined the tasks included in the scenarios and the teaching aids meant to support mathematical discovery. I was also interested in which organizational forms of teaching the students planned to use for this part of the lesson. All the students' scenarios indicated that this part of the lesson was consistently planned as small group work, with groups of 3–4 people. The pupils' job was to work together to solve a task or series of tasks and prepare to present their results to the class. Group work was designed to facilitate pupil discussions, allowing them to exchange individual insights and develop a common position on solving the tasks.

Among the tasks that students proposed, I identified those that foster creativity, i.e., discovery-based tasks, and those that refer to familiar procedures for students to follow. Yeo (2017) classifies tasks of the first type as mathematically rich, as, in his view, they provoke teaching situations that not only require non-schematic behavior but also provide students with the opportunity to learn new mathematical content while developing important processes such as analytical skills, creativity, and metacognition. On the other hand, the procedural tasks included in the students' proposals focused on practicing mathematical proficiency, such as calculus fluency or memorization of an algorithm. Using these tasks, students planned to guide pupils in making generalizations, such as defining the subclass and superclass categories of two familiar concepts (e.g., "Every square is a rectangle, but not every rectangle is a square"). Very often, procedural tasks were preceded by the teacher's presentation of what the students were to discover, so that they would solve the task in the manner it was presented.

In the group of discovery-based tasks, I incorporated those aimed at discovering a rule, property, definition, or relationship between mathematical objects, such as:

> Calculate several sums of any two or three even numbers, then examine the results. What numbers did you obtain? Describe the pattern you observed in your own words (student lesson project).

In the quoted example, the goal is to arrive at the rule that the sum of even numbers results in an even number. The steps toward achieving this goal involve calculating several sums of even numbers and examining the outcomes. However, a more valuable task could be: "Do you believe the sum of any two even numbers will yield an odd number? Justify your response." While the ultimate objective remains unchanged, this task allows for greater creativity in approaching the problem. Another example of a task found in student scenarios is:

> Kasia made birthday party invitations in various polygon shapes and decided to adorn each one with decorative ribbon along the edges. How much ribbon will she require for this task? How did you calculate the necessary length of ribbon? Additionally, explore whether there's a method to determine the required ribbon length for a different set of invitations (Student lesson design).

For this assignment, students planned to utilize the following teaching aids: premade invitations in various shapes including a square, rectangle, equilateral triangle, regular hexagon, quadrilateral, pentagon, and hexagon with different side lengths, all measured in full centimeters (the total sum of the perimeters of these polygons exceeded 200 centimeters). Additionally, a ribbon measuring 200 centimeters in length was to be provided. This approach encourages students not only to assess how much of the ribbon will be used but also to calculate additional perimeters, which is the essence of the lesson that focuses on understanding polygon perimeters.

Unfortunately, across all student scenarios, I found that only 19% of the tasks not only defined the goal of the lesson but also provided a balanced set of cues prompting students to formulate generalizations in the form of new concepts, rules, or properties. I conducted further analysis on this subset of scenarios to assess the extent to which the other stages outlined in the lesson design were implemented. While the initial five stages of the lesson were included in each of these scenarios, two of them failed to prompt students to explore applications for the new discovery. However, I consider this omission a lesser flaw in lesson construction compared to scenarios where the discovery tasks were inadequately designed.

In every lesson plan, no matter the task at hand, the students intended to incorporate various teaching aids. These aids included items like Cuisenaire blocks, decimal tables, Lego blocks, tokens, measuring cups, rulers, geometric figure models, pan scales, and more—all carefully selected to match the lesson's subject matter. The difference lay in how these aids were to be used: during sessions focused on exploration, experimentation, comparison, or model-building, students would have been encouraged to actively use the teaching aids themselves. Conversely, in lessons supposed to revolve around procedural tasks, teachers would likely have demonstrated the use of the aids.

Engaging in a collective discussion about the obtained results is another key aspect of employing this method. This moment of reflection and exchange of new insights should serve as the culmination of the learners' efforts. Among the student scenarios, only two main approaches were observed. The most prevalent involved presenting the group's findings followed by a structured discussion (62%), guided by the teacher's prompts like "Did all groups arrive at the same results? What challenges did you face during the task? Why did you choose this particular approach?" The alternative method also began with result presentations but concluded with the teacher's assessment (38%). This indicates a prevalent traditional teaching perception where the teacher is seen as the primary source of knowledge. Such a method of concluding the discovery process often leads to students merely completing tasks without fully engaging emotionally, which is crucial for meaningful learning. The strategies employed at this stage significantly impacted the subsequent phase—students' independent articulation of knowledge. Surprisingly, 11% of student projects overlooked the importance of developing mathematical language skills at this stage, and in 32%, the new knowledge was articulated by the teacher. A notable shortcoming in many student projects was the assumption that procedural task-solving alone would suffice for learning and conceptualizing new knowledge, evident in 123 scenarios.

Demonstrating the practical application of new knowledge is essential to showcase its relevance. I see this phase as the culmination of the entire discovery process. However, in many cases, the most common approach involved solving textbook or exercise book tasks (24%). These tasks typically revolved around artificial scenarios created solely for teaching mathematical concepts and lacking a connection to real-life situations. More impactful were the students' proposals where the learners themselves constructed examples highlighting the application of newly acquired knowledge (15%) or explored its real-world relevance (9%). Additionally, some lesson plans aimed to prolong student engagement by altering task conditions. For instance, when teaching the commutative law of addition or multiplication, lessons were designed to have pupils test the discovered property with more variables or factors.

Students' opinions on the discovery-based method of teaching mathematical concepts

I was keenly interested in gathering the students' perspectives on the efficacy and appeal of the proposed method for teaching mathematics in early education. The table below illustrates the students' responses to four statements.

Table 2. Students' opinions on the discovery-based method of teaching mathematical concepts

Statement	**Do you agree with the following statement? (data in % rounded to the first decimal place)**						
	definitely yes	**probably yes**	**hard to say**	**probably not**	**definitely not**	**average**	**dev. stand.**
The discovery-oriented teaching method enhances the appeal of math lessons.	57,6	20,7	15,8	6,0	0	4,30	0,94
The discovery-oriented teaching method instills students with confidence in their ability to learn mathematics successfully.	15,8	21,7	32,1	20,1	10,3	3,13	1,21
The discovery-oriented teaching method facilitates better understanding of mathematics.	26,6	60,3	7,6	4,3	1,1	4,07	0,78
Investing time in allowing students to discover mathematics is worthwhile, as I believe thorough explanations alone are insufficient.	20,1	59,2	12,5	4,9	3,3	3,88	0,90

The percentages of student responses in rows 1 and 3 do not sum up to 100, as their values have been rounded to the first decimal place. However, the reliability analysis conducted on student statements indicates a satisfactory level, with the standardized coefficient of Cronbach's alpha at 0.800243 (analysis performed using Statistica 13.1).

Based on the questionnaire responses, we can conclude that that students primarily prioritize enhancing the appeal of mathematics lessons through discovery-focused teaching methods. Furthermore, they believe that this approach will foster a deeper understanding of mathematics among their pupils. However, there is less consensus regarding the perceived benefit of this method in instilling confidence in learners' ability to learn mathematics successfully. This aspect is crucial, especially considering the societal perception of mathematics as a challenging subject that only a few can master. While students recognize the potential benefits of discovery-based learning, such as increased lesson engagement and deeper comprehension, they express reservations about its impact on pupils' self-efficacy in learning mathematics. This hesitation may stem from their lack of prior exposure to discovery-based teaching methods, as indicated by their responses to the question "Have you encountered teaching through the discovery of mathematical knowledge during your lessons?" where the vast majority answered in the negative (83.2% said definitely not; 12% said probably not; and 4.0% said it is hard to say).

Conclusion

The analysis of the collected student scenarios shows that organizing a learning environment using the discovery-oriented method is not an easy task. The primary difficulty highlighted in these scenarios is the struggle to create tasks that effectively inspire students to make their own discoveries—a fundamental aspect of such teaching. Additionally, we cannot label this method as intuitive for two main reasons. Firstly, students' survey responses indicate a lack of prior experience with this approach in their mathematics lessons, leaving them without personal familiarity. Secondly, merely introducing the method theoretically or through occasional participation in demonstration lessons does not ensure success. Therefore, I believe it is valuable to incorporate activities that expose students to unfamiliar teaching methods as part of their college education. This not only broadens their teaching competence but also fosters investigative mindset in mathematics, potentially influencing their mathematical self-efficacy. However, further research and in-depth analysis are necessary to explore this correlation fully.

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