ELEMENTARY EDUCATION IN THEORY & PRACTICE Vol. 19, 2024, No. 1(72) e-ISSN 2353-7787 DOI: 10.35765/eetp.2024.1972.08



Submitted: 3.07.2023 | Accepted: 28.07.2023

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# The Linguistic Picture of Measurement Units in Mathematical Fairy Tales by University Students

Językowy obraz jednostek miar w studenckich bajkach matematycznych

### KEYWORDS ABSTRACT

mathematical fairy tales, textual tasks, conventional units of measurement, unconventional units of measurement

#### In the article, we analyze math problems from mathematical fairy tales by students, created in 2020-2022 at the University of Silesia in Katowice. The aim of the analysis is to identify mathematical concepts and units of measurement and to describe teaching methods suitable for pupils in grades 1-3. We take into account findings from mathematics didactics and methodology, cognitive text analysis theory, neurodidactic guidelines for mathematics education, and the formal requirements for early education. The original research material comes from math problems contained in 98 mathematical fairy tales. We categorize these tasks based on the presence of conventional and unconventional units of measurement. We present those types of metrology tasks that appeared most frequently in the research material. In math problems, the conventional units of measurement are used thoughtfully and intentionally. The presence of unconventional units of measurement often reflects an anthropocentric view of the world. We believe that fairy tales provide an engaging context for early childhood education. An anthology of mathematical fairy tales with teaching suggestions could provide an alternative to traditional school tasks.

VARIA

#### SŁOWA KLUCZE ABSTRAKT

bajki matematyczne, zadania tekstowe, konwencjonalne jednostki miary, niekonwencjonalne jednostki miary

W artykule analizujemy zadania tekstowe pochodzące z bajek matematycznych, napisanych w latach 2020-2022 przez studentów Uniwersytetu Śląskiego w Katowicach. Celem analizy jest wyłonienie pojęć matematycznych i jednostek miary oraz opis rozwiązań metodycznych adekwatnych dla uczniów klas 1-3. Uwzględniamy ustalenia dydaktyki i metodologii matematycznej, kognitywistyczną teorię analizy tekstu, neurodydaktyczne wskazówki dotyczące kształcenia matematycznego oraz wymagania formalne dla absolwentów pierwszego etapu edukacji. Oryginalny materiał badawczy pochodzi z zadań tekstowych zawartych w 98 bajkach matematycznych. W analizie stosujemy podział zadań tekstowych ze względu na obecność w nich konwencjonalnych i niekonwencjonalnych jednostek miary. Prezentujemy te typy zadań metrologicznych, które zyskały największą frekwencję w materiale badawczym. W zadaniach tekstowych konwencjonalne jednostki miary są stosowane w sposób przemyślany i intencjonalny. Obecność niekonwencjonalnych jednostek miary należy wiązać m.in. z antropocentrycznym postrzeganiem świata. Uważamy, że bajki stanowią atrakcyjny kontekst procesu uczenia się dzieci w wieku wczesnoszkolnym. Antologia bajek matematycznych wraz z sugestiami metodycznymi mogłaby stanowić alternatywę dla szkolnych zadań tekstowych.

### Introduction

"The category of quantity stands as one of the fundamental constructs imposed by the human mind onto the perceived world" (Nowosad-Bakalarczyk, 2018, p. 92). The ability to count is a product of civilization, passed down through generations. Counting and measuring make possible cognitive order and a sense of tamed reality (Nowosad-Bakalarczyk, 2018, p. 92). Measurement forms the foundation of scientific inquiry and holds significant importance in education (Jakubiec and Malinowski, 2004; Nawolska and Żądło-Treder, 2020).

In this article, we analyze metrological math problems from mathematical fairy tales, which were created in the 1920s at the University of Silesia in Katowice.<sup>1</sup> Fairy tales can serve as the storyline of the learning process, as they stimulate cognitive activity in young school-aged children (Michalak and Misiorna, 2008, p. 206). "The storyline becomes the central theme and the context of the learning process, within

<sup>1</sup> For a description of the project and an analysis of its results, see previous articles (see Bortliczek and Raszka, 2022, pp. 57–67; Raszka and Bortliczek, 2022, pp. 137–150). The current topic is being addressed for the first time.

which students and teachers seamlessly engage in various activities" (Kosek and Kowalska, 2018, p. 63).

We structured the exemplification material based on Marta Nowosad-Bakalarczyk's classification (2019), which discusses conventional units of measurement (CUM) and non-conventional units of measurement (NCUM), building upon Adam Bednarek's theory (1994). Graduates of the first stage of education are expected to learn CUM as part of the Core Curriculum.<sup>2</sup> In our analysis, we also consider the NCUM which are challenging to categorize, but are part of the linguistic representation of the world. Both of these systems are explored further in the exemplification section.

# Conventional units of measurement in mathematics education

The collected material includes 194 math problems containing CUM. Of these, 155 (out of 66 fairy tales) include questions about time, while 24 involve measuring length. The frequency of problems with CUM is shown in Table 1.

Category	Year/number of tasks			T1
	2020	2021	2022	Iotal
Time measured by clock	13	60	51	124
Time measured by calendar	3	17	11	31
TIME TOGETHER:	16	77	62	155
Length measurements	2	16	6	24
Weight measurements	0	8	1	9
Volume measurements	0	3	1	4
Temperature measurements	0	1	1	2
Money calculations*	8	21	19	48

Table 1. Math problems with CUM

\* Economic problems are not the subject of analysis. Their frequency is cited for comparison.

<sup>2</sup> We use the term *Core Curriculum* to refer to the document: Regulation of the Minister of Education of February 14, 2017 on the core curriculum for preschool education and the core curriculum for general education for elementary school (Regulation..., 2017).

Ultimately, analytical material was selected based on the predominance of mathematical problems necessitating the manipulation of units of time and length.

# Time measured by calendar and clock

As per the Polish Core Curriculum, a third-grade graduate should be able to:

6.4. tell time on a clock with hands and an electronic clock ...; perform simple time calculations; use time units, like day, hour, minute, second; use a stopwatch, phone, tablet, and computer apps; write dates, such as his/her birth ...; navigate a calendar; read and write Roman numerals at least up to XII (Regulation..., 2017, p. 38).

Learning the concept of time involves several key aspects, such as learning about different segments of time (day, week, month, and year<sup>3</sup>) and units of time (day, hour, minute, and second), gauging the duration of time (e.g., how long is an hour) and its flow (Puchalska and Semadeni, 1985, p. 378).<sup>4</sup> Comparing the lifespans of humans and animals contributes to the ability to estimate and calculate age (see [06\_2021],<sup>5</sup> [27\_2021]).

It's going to be quite a memorable birthday since the queen is twice the age of her son, Prince Maximus, who's turning 18. Can you guess which birthday the queen will be celebrating? (task [06\_2021])

She was nine years old. She was called Green Riding Hood. Why? – you will ask. Because she loved the forest, which she took care of as if it were her own home. She often ... helped animals, collected garbage thrown by people, and planted young trees. ... One of her closest friends was a wolf who lived in a massive burrow beneath a century-old oak tree. He was twice as old as Little Green Riding Hood. How old was the wolf? (task [27\_2021])

Using multiplication or division to figure out a character's age, as seen in tasks like [06\_2021] (where the Queen is twice as old as her son Maximus) and [27\_2021] (where the wolf is twice as old as Green Riding Hood), provides a less straightforward approach to the question of how old someone is.

The protagonist of math problem [15\_2021] is eagerly counting down the days until her dream vacation, set to begin on April 1st and last until July 1st.

<sup>3</sup> On learning these concepts, see Puchalska and Semadeni, 1985, pp. 378-379.

<sup>4</sup> The analysis of time-related calculations is undertaken by Puchalska and Semadeni (1985).

<sup>5</sup> In square brackets we give the number of the fairy tale and the year of its creation.

Hania was eagerly anticipating an exciting journey set to begin tomorrow. She had been looking forward to it for what felt like an eternity, ever since April Fool's Day. "Mom, today is the last day of June. Tomorrow, we're finally flying off to our dream vacation. It's going to be amazing!" How long had Hania been waiting for her Italian vacation? (task [15\_2021])

We noted 31 math problems that illustrate the use of the terms "year," "month," "week," "day" and the calculation of the passage of time (for example, how many days it takes to wait for a dream vacation).

The basis for measuring the passage of time is the use of the dial clock model. Initially, clock tasks involve calculations based on full hours (e.g., [15\_2020]), progressing later to involve minutes (e.g., [13\_2020]) or quarters (e.g., [02\_2022]).

Mr. Charles always opens his bakery at 6 in the morning and closes at 5 in the afternoon. How many hours is the bakery open? (task [15\_2020])

The child crosses the twelve-hour threshold by first calculating how many hours pass from six o'clock in the morning to twelve o'clock, and then from twelve o'clock to seven o'clock.

In task [13\_2020], the straightforward question "How many minutes did it rain?" suggests an answer in minutes.

Heavy rain fell from 4:15 p.m. to 5 p.m. After that, a beautiful rainbow appeared in the sky. How many minutes did the rain fall for? (task [13\_2020])

Task [02\_2022] requires crossing the sixty-minute threshold to find out what time Cinderella will arrive at the ball if she leaves at 7:45 pm and the trip takes 45 minutes.

The girl set out for the royal ball at 7:45 p.m. Try to calculate at what time Cinderella arrived at the castle if her journey lasted as long as that of the stepmother and her daughters (task [02\_2022])

In order to determine what time Amelia went on the trip (see [08\_2020]), one must turn back the hands of the clock. This counting strategy is what Puchalska and Semadeni (1985, p. 380) call reversing the problem (in keeping with the fact that time cannot be reversed).

Upon returning to the horse stable, she realized that she had been gone for three hours. Considering it's now 6 p.m., what time did Amelia leave for the trip? (task [08\_2020])

Measuring time using standardized instruments does not guarantee accuracy due to potential clock defects, misreading the hour, or subjective perceptions of time passage (as in "she realized she had been gone three hours"). As Gruszczyk-Kolczyńska (2014, p. 190) points out, "In our perceptions, one hour may not be equal to another" (Gruszczyk-Kolczynska, 2014, p. 190).

# Conventional section measurement

Understanding length measurement involves grasping the fundamentals of measuring and learning about scientific units of measurement. This process typically involves: 1) comparing the lengths of objects by juxtaposing them, 2) employing conventional measurement tools, and 3) using CUM and standardized measurement instruments (Nawolska and Sting-Treder, 2020, pp. 98-104).

In fairy tales, while the events and characters are fictional, the numerical figures and calculations are real. For instance, in task [09\_2021], we find factual information about the actual dimensions of the anteater.<sup>6</sup>

The anteater fascinated the children. The guide explained to them that the anteater in the Ostrava zoo, including its tail, is 2 meters and 20 centimeters long, and its tongue measures 60 centimeters. What is the length of the anteater if you measure it with its tongue extended? (task [09\_2021])

According to the realistic concept of teaching mathematics, math problems should enhance a child's understanding of real quantities, quantitative and spatial relationships, or authentic events (Siwek, 2004, pp. 66–67). In fairy tales, calculations also involve fantasy characters:

Each gnome was 20 centimeters tall, and the castle where the gnomes lived was 3 meters and 20 centimeters taller than the sum of the height of the 14 gnomes. What height was the castle? (task [07\_2021])

The above puzzle requires adding up the height of all the dwarves to calculate the height of the castle.

As noted by Nawolska and Zdzło-Treder, "Tasks that prompt children to estimate dimensions (such as distances) in various settings, including the classroom, playground, and during trips, and then validate their estimations, are valuable." (2020, p. 102). An example of such an activity is the calculations of Goldilocks in the fairy

<sup>6</sup> In this part of the analysis, we base our analysis on 24 math problems present in the 16 fairy tales.

tale [46\_2021].<sup>7</sup> The heroine measures or approximates the height of chairs ("It turned out that the chair measured 165 centimeters"; the other "chair seems to be taller than me by 10 centimeters, and my height is one meter and forty-five centimeters").

A hefty wooden ruler lay on the kitchen counter. Goldilocks resolved to measure the height of the tallest chair. "This chair appears to be the largest," she remarked, eyeing the host's chair. The chair measured 165 cm. Estimating the heights of the other chairs, the girl commented: "The second chair seems to be 10 centimeters taller than me, and my height is one meter and forty-five centimeters." What is the height of the hostess's chair? (task [46/1\_2021])

In contrast, in task [46/2\_2021] Goldilocks compares the result of the measurement with her height ("The host's chair is huge, its height exceeds my height by 25 cm"). Both tasks use a ruler as a standardized tool of measurement ("A hefty wooden ruler lay on the kitchen counter"; "Goldilocks reached for the ruler again").

The golden-haired girl reached for the ruler once more to measure the height of each chair. "The host's chair is huge, its height exceeds my height by 25 cm! But it is very uncomfortable." How high is the host's chair? (task [46/2\_2021])

Learning the concept of length measurement begins with comparing one's own height with that of peers and other objects (Nawolska and Sting-Treder, 2020). Mateusz Adamczyk illustrates the evolution of this competence:

A thing cannot be big by definition and maintain this property regardless of the context, e.g. an elephant is big compared to a dog, but when you put it next to a pyramid, it will be small. In the early years of life, children tend to attribute absolute meaning to such words (2019).

The child gains mathematical experience through measuring distances and representing them through drawings, maps, or written records of their actions. Solving math problems should incorporate practical connections to real-life situations. For instance, Alma the frog takes a route measured in meters when shopping.

Alma's backpack was filled to the brim as she set off from the decoration store. On her way back, she passed the "Lake" supermarket and finally the vegetable store. How many meters did Alma carry this heavy backpack if the distance between each store ... is 200 meters, and from the vegetable store to her house is also 200 meters? (task [31\_2021])

<sup>7</sup> Quoting two tasks from this tale, we use the designations [46/1\_2021] and [46/2\_2021].

Task [31\_2021] provides all the necessary information to answer the final question ("How many meters did Alma carry that heavy backpack ...?"). A map illustrating Alma's neighborhood can aid in the calculation.

Tens of kilometers distance is traveled by the characters of the fairy tale "The Śmiechołki Family" during their trip to the zoo (see [20\_2021]).

The distance from the Śmiechołkis' home to the zoo is 94 km. Dad stopped for a break after driving 50 km. How many kilometers are they from their destination? (task [20\_2021])

The result is obtained by subtracting the distance already traveled (50 km) from the total distance of the route from home to the zoo (94 km).

Learning the concept of perimeter involves measuring a segment and summing up the lengths of the sides of a polygon: objects in the real world or drawing models. Perimeter is expressed as a measure of continuous size with fixed units (Nowik, 2009, p. 141; Nawolska and Sting-Treder, 2020, pp. 104–105). In terms of understanding geometric concepts, the primary school curriculum notes that a third-grade student should:

5.2. measure lengths of segments and sides of geometric figures, etc.; provide the result of the measurement, use units of length, such as centimeter, meter, millimeter; explain the relationship between different units of length; use binomial expressions; explain the concept of kilometer;

5.3. measure the perimeters of various shapes using measuring tools, including in reallife contexts; calculate the perimeter of a triangle and rectangle (including a square) with given sides (Regulation..., 2017, p. 38).

In task [07\_2020], only the length of the netting needed to fence the plot is given. However, a complication arises as the heroes end up buying too much netting.

Dad calculated that he needed 12 meters of netting to fence the plot against intruders, but mom bought twice as much because she didn't write down dad's measurement. "No problem," dad said, and he simply used as much netting as he needed. The question is: how many meters of netting are left after fencing the plot? (Task [07\_2020])

A similar scenario is used in task [47\_2021].

In the garden, the elves are planning to construct 3 identical flower beds using wire mesh, with one side of each square flower bed measuring 2 meters. They intend to plant tulips of different colors in the center of each flower bed. How many meters of wire mesh will they require to build all 3 flower beds? (Task [47\_2021])

Task [47\_2021] centers on squares and requires calculating the total length of wire mesh needed for the gardening project.

As the class ventured deeper into the zoo, they came across a sizable lion enclosure. The enclosure for the king of animals formed a rectangle measuring 15 meters by 20 meters. In contrast, the tiger's enclosure was more compact. Each side of the tiger's enclosure was 5 meters shorter than the corresponding side of the lion's enclosure. What is the perimeter of the tiger's enclosure? (task [09\_2021])

Task [09\_2021] revolves around rectangles, specifically comparing the lion and tiger enclosures, both of which have this geometric shape.

The dense forest surrounding the castle was bordered by a rectangular defensive wall. One side of the wall measured 10 kilometers, while the other measured 8 kilometers. What was the total length of the entire wall? (Task [07\_2021])

The question at the end of task [07\_2021] seeks the length of the wall in kilometers. These tasks serve to reinforce the concepts of meters and kilometers, demonstrate the practical application of measurement skills, teach the importance of rationalizing results, and assess the accuracy of calculations. In today's world, precise measurement is considered "an essential component of science crucial for the advancement of civilization and technological progress" (Nawolska and Sting-Treder, 2020, p. 8).

# Unconventional units of measurement in teaching mathematics

Words that denote a container or its contents serve as units of measurement for substances. These terms, referred to as NCUM, encapsulate both the concept of the container itself and the quantity of matter it holds. Their range is extensive and continually evolving (Nowosad-Bakalarczyk, 2019, p. 107). As human societies progressed beyond the limitations of measuring with body parts, the use of containers became essential (Nawolska and Sting-Treder, 2020, p. 8).

containers of a fixed shape and size began to be used for measurement, in which liquid and loose substances were placed (... that easily filled the space of the container) (Nowosad-Bakalarczyk, 2019, p. 105).

Portions measured with NCUM

can undergo further mental (arithmetical) operations to further specify the amount of matter that is being assessed, as expressed by expressions with the numerator (two bowls

of broth, five jugs of water were poured into the boiler, half a barrel of wine) (Nowosad-Bakalarczyk, 2019, p. 107).

The treatment of substance as a morphic object is closely tied to the physical characteristics of the container and its usage. This analysis reveals that

the units of measurement become the containers in which a substance is ... customarily made (pot), stored (barrel), transported (watering can), sold (bottle) or served (if it is intended for consumption, such as a glass) (Nowosad-Bakalarczyk, 2019, pp. 107–108).

Packaging ... allows the boundaries of something to be clearly marked – the objects used for this purpose enable man to establish the boundaries of entities conceptualized as substances and thus give them the attributes of tangible entities that can be quantified (two bags of potatoes) (Nowosad-Bakalarczyk, 2019, p. 109).

Packaging forms a set of containers. NCUM Nowosad-Bakalarczyk includes also non-containers in this group:

The amount of matter can also be determined without reference to another object, as seen in phrases like a slice of bread, a bar of chocolate, a head of lettuce, a bunch of chives, a grain of sand, a pair of tights, a piece of meat, and a herd of horses (Nowosad-Bakalarczyk, 2019, p. 109; after Bednarek 1994).

Non-containers are combined with morphic nouns (e.g., a specimen of a butterfly, an issue of a newspaper, a head of cattle, a pair of pants, a piece of underwear, a collection of reports) and non-morphic nouns (e.g., a nugget, a head, a lump, a clump, a cob, a leaf, a tooth, a grain, and others) (Nowosad-Bakalarczyk, 2019, pp. 109–110). Juxtaposed with non-morphic nouns, non-containers are divided into natural units (see the examples mentioned above) and artificial units (e.g., sheet, bale, loaf, lump, scoop, tuft, cube, slice, ball, heap, roll, sheaf, stack, pile, spool, piece, slate, and coil) (Nowosad-Bakalarczyk, 2019, p. 111).Classification of NCUM math problems is difficult because:

(1) CUM and NCUM can occur within a single task, and unconventional units of measure can be converted like conventional units (or vice versa), as in the following task:

The horse lived in the stable, the calves in the barn, the chickens in the coop and the rabbits in a cage. Tom prepared 20 kilograms of food for the animals. He divided the food into 4 equal portions. How many kilograms of food went to each of the enclosures? (task [45\_2021])

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2 The operation of the open set of NCUM is cultural (non-scientific, practical). The existence of NCUM is often disregarded in early childhood education curricula. The act of measuring using body parts is typically considered only in preschool education curricula:

IV. A child being prepared to enter school should: ...13) experiment, estimate, predict, measure the length of objects, using, for example, a hand, a foot, a shoe (Regulation..., 2017, pp. 6–7).

3 The number of fairy tales containing NCUM is not representative (in comparison to those featuring CUM), which is due to the focus on the teaching of CUM in mathematics education.

To illustrate the presence of NCUM in mathematical fables, we quote selected tasks. The cited examples are not analyzed, but we highlight the presence of NCUM in the questions (scoop, portion, basket, and jar). We plan to explore this topic further in a separate article.

Rapunzel treated her family to artisanal ice cream. Jacek and Filip ordered a double scoop, while Grandma Asia and Aunt Kleopatra each chose a single scoop of Advocate flavor. How many scoops of ice cream did the cousins, aunt, and grandmother enjoy together? (task [33\_2021])

Mr. Charles uses one portion of marmalade to fill 10 doughnuts. If he has 30 doughnuts to fill, how many portions of marmalade does he need? (task [15\_2020])

After lunch, Grandpa Marian took his grandchildren to the garden to pick fruit. Kasia gathered 9 baskets of raspberries and 3 baskets of blueberries, while Marcin harvested 15 baskets of gooseberries and 5 times fewer baskets of blueberries. How many fruit baskets did the children collect altogether? (task [35\_2021])

The next day, Snow White not only cooked dinner but also cleaned out the pantry. She discovered that several jars of vegetables and fruits had expired. Snow White discarded 5 jars of cucumbers, 2 jars of strawberry jam, 3 jars of apple compote, and 6 jars of plum jam. How many jars did Snow White throw away? (task [26\_2022])

Mathematics is a subject deeply intertwined with human cognitive activity. The role of educators is to understand the determinants of the teaching-learning process within mathematical methodology. Anna K. Zeromskaya emphasizes the importance of humanizing the perception of mathematics within the school environment and mitigating its negative societal connotations:

The anthropomathematical approach centers on the phenomena that accompany the cognitive process, where the subject of cognition is the human being (*anthropos*), and the object of this cognition is mathematics (*mathematika*) ... Anthropomathematics explores all phenomena closely linked to the nature of mathematics as an object of cognition, which do not exist autonomously and independently of the cognitive subject, the human being. (2013, p. 24)

Task [10\_2020] from the tale "Winnie the Pooh's Ordinary Day" aligns with the cultural framework of human life:

Pooh knocked on the door of Christopher's house a while later. The boy opened the door and sad in a sad voice:

I can't play with you now, Pooh. I have to do my homework.

Hearing this, Pooh sighed loudly and sat down by the door, visibly disheartened. Christopher became very sad at the sight of the forlorn bear, so he also sat down and embraced Pooh.

- Once you're out of school, I'll come to see you every day, Christopher! announced Pooh in a serious tone.

– But Pooh, when I finish school, I will have to go to work... – replied Christopher. Then Pooh got sad and asked:

– And where will you have to go when you finish work? Christopher chuckled, replying: – Silly bear. When you finish work, you retire and nothing stops you from having fun anymore. Only... Only it will be many years before I retire, because I will be 65 then. Furrowing his brow in thought, Pooh asked, and after a moment asked:

- And how old will I be then? The boy smiled and said:

- 64, Pooh.

By how many years is Winnie the Pooh younger than Christopher? (task [10\_2020])

In this NCUM story, there's a chronological sequence depicting different stages of a person's life: 1) schooling ("I have to do my homework"); 2) work activity ("When I finish school, I'll have to go to work"); 3) retirement ("When you finish work, you retire"). The final question about Winnie the Pooh's age ("How many years is Winnie the Pooh younger than Christopher?") is set in the biographical context of Alan A. Milne and his son Christopher R. Milne.

# Applications

The ability to measure and operate CUM are important skills that children learn at school. The progression in mathematical competence and understanding of units of measurement moves from handling NCUM to handling CUM. The learning of these units is closely related to the acquisition of language (from everyday to scientific and

specialized concepts). The occurrence of NCUM in the cultural code should be used extensively in child education under the anthropomathematical approach:

In the anthropomathematical approach, we view mathematics as an intellectual activity rather than a set of "ready knowledge." We regarded as both an object and a result of cognitive processes inseparably connected with a person – an actively cognitive individual. This approach recognizes and highlights the fact that cognitive processes occur within specific social contexts. We are keenly interested in the widely understood interdependence between mathematics and the human learner. This is, after all, precisely the nature of educational processes (Żeromska, 2013, pp. 22–23).

NCUMs find their place in discourse where precision of expression is prioritized, such as scientific, official, and educational contexts. In colloquial discourse, however, NCUMs like "mug" are understood based on their established shapes and volumes, with minor variations between individual instances being negligible. In everyday communication, we use NCUM more often than CUM. This underscores the importance of initially introducing NCUMs in mathematics education, gradually transitioning to CUMs. Such an approach aligns with children's familiarity with the decimal counting system and is consistent with the assumption that mathematics is deeply intertwined with human cognitive activity.

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