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Creation of abstract geometric concepts through both child's creativity and creative activity of a teacher

One of more important areas is geometry. Students can to create of abstract geometric concepts through both child's creativity and creative activity of a teacher. I chose the topic connected with early math learning because this subject is very important to get the necessary skills to learn. Early math skills predict later mathematics and later reading. This is an important mathematical idea that the kids form at the early age and it helps them learn to learn.

D. Clements, lecturer and instructor, (who teaches at the Buffalo University in New York), enumerates three compontents which are the most important in math teaching: the first is mathematical goal, the second one is the development of sequence of levels of thinking and the third one is corelated instructional enviroments and specific activities in which we want kids to engage to achieve the next level of thinking1. He claims that everybody can learn math but there is one group of people who are good at math and the other one that are not so good at math. He

[1] P. Daro, F.A. Mosher, T Corcoran, *Learning Trajectories in Mathematics: A Foundation for Standards*, Curriculum, Assessment, and Instruction, Philadelphia 2011, p. 23-24.

believes the most important thing is to give children a good opportunity to learn math. When do we give this opportunity to our children? When do we become the true professionals?

Teachers in everyday work with children should try to use such components as: knowledge, skills, previous experience, observation of a child, reflection and finally creativity. I think that we can be the true professionals when we realize what we want to teach children and what level they represent. This knowledge determines possibile ways to achieve the intentional aim. I would like to write about these three components of math teaching.

I. The first one is the development of sequence of levels of thinking.

Everybody knows that a triangle, a rectangle, a line are abstract terms. In real life children deal with for example a box, a ball, a brick. In these objects children detect not only properities of shapes but also size, texture.

There is a special way to geometric understanding. The most important thing is child's perception, but it is worth remembering that geometric world can't be perceived directly. It is hidden in the real world and emerges thanks to special intellectual activity known as geometric intuition[2].

Thinking about each level involves execution of different kind of activities (manipulation, drawing, description). If we want to make the whole process of interiorisation work, we should know that children first have to manipulate with something, then draw and finally describe something. J. S. Bruner calls this sequence: levels of representation.

The first one is an enactive level of representation. Children do something, they know the whole process of this activity and they can repeat it. The whole sequence of these actions defines the way to cope with the situation.

The next one is an iconic level of representation. Pupils on this level show us their knowledge about an object through a picture. On the basis of the drawing we can learn what children know about things.

And the last one is a symbolic level of representation. People on this level describe objects using a symbolic code for example parlance (a phraseology, expressions used by a particular group of people), symbolic language and so on[3].

Each representation defines a different way to cope with the input (new information). We seldom deal with only one representation. Usual-

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[2] E. Swoboda, *Regularności geometryczne w uczeniu się dzieci*, [w:] *Dziecko i matematyka*, red. J. Gunèaga, E. Swoboda, Rzeszów 2009, p. 47.

[3] G. Treliński, U. Trelińska, *Kształtowanie pojęć geometrycznych na etapie przeddefinicyjnym*, Kielce 1996, p. 15.

ly, they have composite character, for example iconic-symbolic, symbolic-enactive and so on. Child's learning process is based on learning each level of representation in the correct order (which is coping with something through manipulation, using pictures or description) with a simultaneous transition to the next level and numerous returns to the previous one. Each return makes the activity and picture representation more profound than original rules and makes the development of the learner's language (symbolic code) faster and fuller.

Other researchers P. van Hiele and D. van Hiele-Geldof proposed a model of five levels of geometric thinking[4]

1. The lowest level is a visual level. Pupils recognize and identify two-dimensional shapes and three-dimensional figures by their appearance as a whole (without emanating particular properties). The child being on this level will say, for example: This is a square, but when we ask him: why?, he can't say. He will say: it looks like a box.

2. The second level is a descriptive level. At this level a pupil analyses known figures and he can describe their properties. For example, a student will talk about equilateral triangle. He notices that this figure has such properties as three sides, all sides equal, three equal angles and symmetry, both line and rotational. At this level figures are described on the basis of their properties which students detect during many different experiences. These properties are not logically ordered yet. At that time, a very important thing is language.

3. The next one is an informal deduction level. At this level student orders all properties of figures in a logical way and can use them to justify relationships. For example, students can explain why all squares are rectangles.

4. The fourth level is a deduction level. Students use deductive reasoning to draw conclusions about abstract geometric principles. Language at this level has an abstract character and only at this time students can understand the meaning of definitions, the role of axioms, theorems, and their converses.

5. The last one is a rigour level. Students compare different geometric theories and hypotheses. This high level requires advanced mathematical thinking.

Why this knowledge is so important for us? Pierre M. van Hiele, a former Montessori teacher argues that teaching school mathematics – geometry and arithmetics – has been a source of many misunderstandings. It

[4] P.M. Van Hiele, *Developing Geometric Thinking through Activities That Begin with Play*, "Teaching Children Mathematics" 6(February 1999), p. 310-316.

is connected with incoherence between students level of thinking and instruction that promotes reasoning about geometric ideas. For example, when we talk about square we can spot different meanings among children representing different levels of thinking. For a student at the visual level, squares may mean a variety of shapes that "look like a perfect box" no matter which way they are rotated. For students at the descriptive level of thinking, a square is a closed figure with four equal sides and four right angles. But even to these pupils , a square has no relationship to the class of rectangles, as it does for students at a higher level[5]. In the book entitled "A guide to effective instruction in mathematics" we can read: "although most levels in the model of geometric thinking do not pertain to students in Kindergarten to Grade 3, it is important for teachers of primary grades to consider the following:

• Progression from one level to the next one is less dependent on students' age or maturation than on instruction that promotes reasoning about geometric ideas. Teachers of primary students need to provide the kids with instructional activities that help students move beyond merely recognizing two-dimensional shapes and three-dimensional figures (which is level 0) to understanding the properties of shapes and figures (which is level 1).

• The levels are sequential, and success at one level depends on the development of geometric thinking at the preceding level. If students' level of thinking does not progress beyond level of visualization, it is likely that they will have to struggle with geometric concepts at higher levels"[6].

If we analyze these theories in depth we can find connections between them. We can say that developing each level of thinking and providing appropriate language for this level depend on three basic levels of representation.

What is more, D. Clements adds that these levels can help us understand how children think about shapes and conversance of these levels can also guide teachers to provide appropriate learning opportunities for children.

II. More important thing than developmental level are opportunities to learn about shapes.

When can we create good opportunities to learn geometry? To respond to the question we need to know mathematical goal that we want kids to be able to achieve at any given age. Now let's move on to the next component. Teachers need profound understanding of the mathematics in which they want kids to engage. What is important about geometry? Let's try to unpack content knowledge about geometry.

[5] D. Clements, *Young Children Idea about Geometric Shapes*, "Teaching Children Mathematics" 6(April 2000), p. 482-488.

[6] *A guide to effective instruction in mathematics. Kindergarten to Grade 3*, *Geometry and Spatial Sense*, Ontario Education excellence for all, 2005, p.12.

A practical application of the principles and theories behind good instruction are presented in the book 'A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 3'. In this approach, knowledge and skills about geometry are divided into three parts and they concentrate on mathematical concepts of Geometry and Spatial Sense. They are called "big ideas". The "big ideas" in Geometry and Spatial Sense are the following:

▶ Properties of two-dimensional shapes and three dimensional figures

What should we know about properties of two-dimensional shapes and three-dimensional figures in the primary grades:

- Two-dimensional shapes and three-dimensional figures have properties that allow them to be identified, compared, sorted, and classified.
- Experience with two-dimensional shapes and three-dimensional figures, represented in a variety of forms, sizes, and orientations, allows students to understand those properties7.

Geometric relationships

We need to know about geometric relationships in the primary years that:

- Two-dimensional shapes and three-dimensional figures can be composed from or decomposed into other shapes and figures.
- Relationships exist between two-dimensional and three-dimensional geometry (e.g., the two-dimensional faces of three-dimensional figures).
- Relationships exist between categories of two-dimensional shapes (e.g., rectangles are also quadrilaterals, squares are also rectangles).
- Congruence is a special geometric relationship that is shared by shapes having the same shape and the same size[8].

▶ Location and movement

The following are key points that can be made about location and movement in the primary years:

- The location of an object can be described in terms of its spatial relationship to another object or in terms of its position on a grid.
- Transformational geometry involves translations (slides), reflections (flips), and rotations (turns).
- Symmetry can be used to analyse and create shapes in which one half is a reflection of the other[9].

D. Clements says that when we think about geometric aims of teaching we should: reconsider teaching "basic shape" only through examples, give children credit for what they know, avoid common misconceptions, expand the limited notions that are "taught" too often, match activities to children's level of thinking about shapes.

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[7] Ibidem, p. 7-
[8] Ibidem, p. 25.
[9] Ibidem, p. 43.

Many geometric skills develop simultaneously with geometric concepts. Each of them is connected with certain actions – activities which are presented when a child deals with objects and geometric situations. These geometric activities are the following[10]:

- Observation
- Manipulation
- Research
- Description
- Construction
- Creativity

During learning and teaching process these activities do not appear alone but they interpenetrate and complement each other. Having been given good opportunities, students should discover geometry. What should appear in the learning process is intuitive experimental activities. Taking into consideration all the aspects that I mentioned it is important to know that only having knowledge about mathematical goal and the development of sequence of levels of thinking we can think and talk about the child's creativity and creative activity of a teacher.

III. Possible ways of creating abstract geometric concepts through child's creativity and creative activity of a teacher.

P. van Hiele claims that "to promote the transition from one level to the next, instruction should follow a five-phase sequence of activities. Instruction should begin with an inquiry phase in which materials lead children to explore and discover certain structures. In the second phase, direct orientation, tasks are presented in such a way that the characteristic structures appear gradually to the children, for example, through puzzles that reveal symmetry of pieces or through such games as "feel and find the shape." In the third phase, explicitation, the teacher introduces terminology and encourages children to use it in their conversations and written work about geometry. In a fourth phase, free orientation, the teacher presents tasks that can be completed in different ways and enables children to become more proficient with what they already know, for example, through explorations of making different shapes with various pieces or through playing clue games. In the fifth and final phase, integration, children are given opportunities to pull together what they have learned, perhaps by creating their own clue activities. Throughout these phases the teacher has various roles: planning tasks, directing children's attention to geome-

[10] J. Nowik, *Kształcenie matematyczne w edukacji wczesnoszkolnej*, Opole 2011, p. 150.

tric qualities of shapes, introducing terminology and engaging children in discussion using these terms, and encouraging explanations and problem-solving approaches that make use of children's descriptive thinking about shapes."[11]

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Summary

The child during its first years of life operates with three-dimensional items from real world. There is a moment in the educational process when its activity takes place in the dimension of the plain. Functioning at the plain requires understanding the concepts and manipulating specific properties (features) of geometric terms. Supporting the child in this area requires from the teacher to many activities that will produce in a child's mind more and wider understanding of concepts such as line, triangle or cube.

Child creativity present in education brings joy of creating, positive emotions, natural ability of testing, experimenting, drawing conclusions and comparing. Creative mathematical activity? What opportunities and benefits does it bring? What is it necessary in the development of geometric concepts and skills?

[11] P.M. Van Hiele, *Developing Geometric Thinking through Activities That Begin with Play,* "Teaching Children Mathematics" 6(February 1999), p. 316.

Keywords: abstract geometric concepts, child's creativity, two-dimensional shapes, three dimensional figures, geometric relationships, geometric activities, shapes.

Budowanie abstrakcyjnych pojęć geometrycznych poprzez twórczą aktywność dzieci i nauczyciela...

Streszczenie

Dziecko w pierwszych kilku latach życia operuje trójwymiarowymi przedmiotami realnego świata. Jednak przychodzi taki moment w edukacji, gdy jego aktywność zostaje sprowadzona do działań w wymiarze płaszczyzny. Funkcjonowanie na płaszczyźnie wymaga rozumienia pojęć i operowania specyficznymi własnościami figur geometrycznych. Wspieranie dziecka w tym obszarze wymaga od nauczyciela wielu zabiegów, które pozwolą wytworzyć w dziecięcym umyśle coraz bogatsze rozumienie takich pojęć, jak prosta, trójkąt czy sześcian. Kreatywność dziecięca obecna w edukacji niesie z sobą radość tworzenia, pozytywne emocje, naturalną możliwość sprawdzania, eksperymentowania, wyciągania wniosków i porównywania. Twórcza aktywność matematyczna? Jakie daje możliwości i korzyści? Po co jest potrzebna i konieczna w kształtowaniu pojęć i umiejętności geometrycznych?

Słowa kluczowe: abstrakcyjne pojęcia geometryczne, twórcza aktywność dziecka, figury płaskie, bryły geometryczne, relacje przestrzenne, aktywności geometryczne, kształty.