God as Absolute Machine: Aligning Modern Formalisms to Prove God

Ward Blondé

ABSTRACT Let Anselm's God denote that than which nothing greater can be conceived. The rationale of this paper is to show that not only the existence, but also three omni-attributes of Anselm's God—omnipotence, omniscience, and omnipresence—can be defined and proven via modern formalisms. The objective is to do this via a terminological alignment of set theory, mereology and computer science on the one hand, and metaphysics and natural theology on the other. The methodology used consists of a two-step argument: first, if physical entities are of paramount ontological greatness, then God is equal to an absolutely infinitely large, physical universe with omni-attributes. Second, using a slightly different criterion, God can be either abstract, or concrete and non-physical. Some important findings are that (1) a central axiom explains both God and the physical realm, (2) Cantor's Absolute Infinite—and therefore God—can be given a consistent definition, and (3) isolated possible worlds are never observed. The essence of this paper is, in short, that "God is the Absolute Machine."

KEYWORDS Anselm's ontological argument; computer science; Cantor's Absolute Infinite; natural theology; proof of God; set theory.

Acknowledgments:
I thank Emanuel Rutten for the suggestion to investigate infinities, Ludger Jansen for his many suggestions and discussions about sections of the paper, and the anonymous reviewers who gave feedback on earlier versions of this paper.

✍ Ward Blondé, Ghent University, Belgium ✉ wardblonde@gmail.com iD 0000-0001-7624-4069

FORUM PHILOSOPHICUM 30 (2025) no. 2, 65–86 SUBM. 6 April 2025 ACC. 25 July 2025
ISSN 1426-1898 E-ISSN 2353-7043 DOI: 10.35765/forphil.2025.3002.04

1. INTRODUCTION

Anselm of Canterbury defined God as "that than which nothing greater can be conceived" (Marenbon 2015). Starting from this definition, he argued that God exists in reality. Many philosophers, such as Aquinas, Descartes, Leibniz, Kant and Plantinga, have commented on his ontological argument, which originated in the 11th century (Oppy 2018).

The aim of this paper is to go further than Anselm's existential proof and demonstrate that God, so defined, is omnipotent, omniscient, and omnipresent, by using modern formalisms. More precisely, the research objective is to establish proof-enabling definitions of God and God's omni-attributes that are rooted in a merger of set theory, mereology and computer science on the one hand, and metaphysics and natural theology on the other.

The methodology consists of providing a philosophical justification of a central axiom that underlies the proof of God, followed by the proof in two steps. The first step of the latter is to prove that God is equal to an absolutely infinitely large physical universe, and that this physical universe is omnipotent, omniscient, and omnipresent.[1] This proof makes use of a criterion that takes physical entities to count as ontologically greater than non-physical ones. However, altering this criterion so that abstract entities, or concrete non-physical entities, are held to be ontologically the greatest, is also investigated. This issue is considered in the context of the second step of the proof.

The central axiom essentially states that there are as many abstract entities as physical entities: namely, absolutely infinitely many. This means that the ontologies of the five formalisms (from set theory to natural theology) all have the same structure, and all have absolutely infinitely many elements. Because of this, their ontologies can be aligned. Using these formalisms, the essence of this paper can be expressed in a single short sentence: "God is the Absolute Machine."

Apart from the question of whether God and God's omni-attributes can be proven through modern formalisms, the paper addresses the following research questions: (1) Why is there something rather than nothing? (2) Are God and the Absolute Infinite consistent? (3) Which formalisms can prove God and render Him conceivable? (4) Is axiological greatness of superior importance to mereological greatness? and (5) Can set-theoretic realism be reconciled with the observations of the empirical sciences? The last of these questions also represents the most significant research gap in this paper, but can

1. Cantor introduced the Absolute Infinite and associated it with God (Jané 1995). He believed it would be possible to prove that the universe was finite and distinct from the mind of God, which he believed to be absolutely infinite. He never proved these things, however.

be answered, in my view, by appealing to evolutionary conservation (Blondé 2016, 2019) in the context of cosmological natural selection (Smolin 1992).

The next section contains a literature review. Section 3 expresses the technical definitions and the central axiom in the working theory used in the first step of the proof. The methodology in Section 4 provides a philosophical justification of the central axiom and the proofs of God and God's omni-attributes. Three important findings are presented in Section 5. A discussion follows in Section 6 and the conclusions are summarized in Section 7.

2. Literature Review

According to Anselm, a non-theist can always conceive of God in their mind. However, a god that exists in reality is even greater than a god that exists only in the mind. Therefore, God, the greatest conceivable being, exists in reality. Kant objected to this argument by appealing to the claim that existence is not a real predicate, meaning that it is a second-level property that cannot be essential with respect to anything (Forgie 2000). This undermines the ontological argument.

Geach's (1956) account of intentional comparatives shows that a phrase like "greater than" does not require either of the entities being compared to actually exist in reality and/or be great. Thus, "that than which nothing greater can be conceived" may function merely within the conceptual space of intentional comparison, without implying greatness of existence in reality. Nevertheless, consider the following two definitions:

1. "that than which nothing greater can be conceived"
2. "that which is greatest"

The greatest entity of (1) requires maximum proof-theoretic strength—at least if we are using theories with axioms and definitions to conceive of entities. The greatest entity of (2), on the other hand, can be interpreted as trivially existing in any theory, weak or strong, that has a greatest entity. As such, it may be the big bang universe, or the smallest transfinite ordinal ω. For this reason, Anselm's definition (1) will be used in this paper. While Kant's and Geach's arguments may very well prohibit Anselm's leap from conceptual to real existence, the proof in this paper does not depend on this. Instead, the proof stands or falls with the central axiom, which is defended via a form of reasoning that is clearly distinct from that of Anselm.

Computer-scientific theories that have similarities with the argument in this paper are those of Jürgen Schmidhuber (2006), Max Tegmark (2008), and Eric Steinhart (2010). The Great Programmer of Schmidhuber runs a program that computes all the computable programs on a universal Turing machine (having memory and available time equal to ω, the smallest infinity).

Our observable universe, which is describable by a finite string of bits S, is computed by one of these Turing-computable programs. However, Blondé (2015) shows that, based on equal probabilities, the program that computes our observable universe computes it infinitely many times. If non-halting programs are banned, then still the program that computes us does not start after any finite time. Consequently, even though S is Turing computable, running the program that computes us requires at least transfinite resources (beyond ω). If, on the other hand, the first string S is chosen that is compatible with our existence, then this will be the execution of a program P that deals with resource limitations. P will, therefore, result in a small universe with a low resolution, or even an artificially intelligent mind that is just intelligent enough to count as being compatible with our existence.

Tegmark augments his Mathematical Universe Hypothesis with the Computable Universe Hypothesis. The first hypothesis is that all structures that exist mathematically also exist physically, and that our observable universe is such a structure. The second hypothesis says that the universe is brought about through Turing-computable functions. Just like Schmidhuber, Tegmark puts a severe limitation on the computational power of what computes the universe. Nevertheless, ordinal machines with memory size beyond that of a Turing machine (this being, therefore, a transfinite size) can compute finite answers to questions that cannot be computed by a Turing machine. Such ordinal machines will inevitably favor or disfavor the abundances of finite worlds that contain a correct or a false answer to questions that are not Turing computable. Moreover, transfinite ordinal machines produce many (infinitely) more worlds than Turing machines. This suggests again that the memory and available time of the ordinal machine that computes the world must be far beyond the countable ω.

According to Steinhart, we live in the first level of absolutely infinitely many (or Ω-many) levels of simulations in simulations. God can then be found at level Ω. This theory comes close to the metaphysics that is proposed in this paper—especially because it uses Ω levels instead of only ω. Two differences stand out: first, Steinhart restricts reality based on the requirement that it consists of levels that simulate other levels through civilizations with computers, and second, he has difficulties explaining why there is something rather than nothing.

3. Theoretical Framework

In order to define and prove God's existence and attributes, a foundational framework of definitions is called for within the formalisms furnished by set theory, mereology, metaphysics and computer science. All these formalisms can be translated into formal theories that have a number of entities that can be extended from natural numbers to ordinals and the Absolute Infinite. For this reason, set theory is introduced first, as this is the canonical basis for such formal theories. The central axiom can be found in the subsection dealing with metaphysical definitions.

3.1. Foundational Set-Theoretic Definitions

For the purpose of introducing set theory, some theory-related terms will first have to be defined. A sentence will consist of a sequence of symbols from an alphabet, constructed according to the syntactic rules of a language. A collection of sentences will be a theory. A theory will have a model if and only if (henceforth iff) there exists a structure (called a 'model') in which all the sentences of the theory are true. A theory will consist of axioms and definitions (sentences assumed as starting points), and further sentences (theorems) will be deduced from them using rules of inference. A theory will be formal iff all its sentences are recursively enumerable by a Turing machine.[2] An entity E exists theoretically (or proof-theoretically) according to a theory T if the existence of E can be proven in T.[3]

The axioms of the theory ZFC (Zermelo–Fraenkel set theory with the axiom of choice) (Zermelo 1908) are about sets in V, the universe of all the pure, well-founded sets, which is known as the 'von Neumann universe' (Neumann 1928). ZFC can be strengthened by infinitely many formal, relatively consistent axioms about sets that can neither be proved, nor disproved by ZFC. Any such extension is a formal theory. The weaker theories, with less axioms, are fragments of the stronger, more extended (or more expressive) theories. The theory that extends ZFC with absolutely all the true axioms, jointly making up the largest possible V, will be called ZFC^{Ω}, and is not formal. The definition of V via ZFC^{Ω} implies two things: (1) any formal theory T, set-theoretic or otherwise, can be translated into ZFC^{Ω} via an injection[4] in V of the collection of entities that exist theoretically according to T, and, conversely, (2) every set in V exists theoretically according to some formal theory.[5]

2. This means that, in formal theories, all the axioms, theorems, symbols, and syntactic rules must be recursively enumerable.

3. Theoretically existing entities do not necessarily exist in reality. All set-theoretic talk about existence is considered to be about theoretical existence, while existence in reality is a first-level property that is a subproperty of the second-level theoretical existence (Forgie 2000).

4. An injection will be an all-to-some relation that works via one-to-one links.

5. Note that if this were not the case, we could construct a theory P in which the smallest set x that does not exist theoretically according to any formal theory exists theoretically

Ordinal numbers include the natural numbers, but also the extensions of the natural numbers beyond ω, the smallest infinity (Cantor 1883). Ω, the Absolute Infinite, exceeds every formally definable (set) ordinal and is itself a proper class ordinal. More technically, Ω is the proper class cardinality of the non-formal class

$$C=\{S_v \mid v \in M\}$$

where:

- the set-theoretic multiverse M is the non-formal proper class of all universe models v of any formal extension of ZFC (Hamkins 2012),
- for each $v \in M$, S_v is a set such that $S_v \in v$, and
- $v \mapsto S_v$ selects exactly one set S_v from each universe model v.

What is innovative about ZFC^{Ω} is that the number of ZFC^{Ω} axioms is equal to Ω and can, therefore, not be formally defined.[6] Because its axioms cannot be listed by an effective procedure, ZFC^{Ω} escapes Gödel's (1931) incompleteness theorems: it can be both consistent and complete, and it can prove all the arithmetical truths.

The theory NBG (von Neumann–Bernays–Gödel set theory) (Neumann 1928) can also be extended to NBG^{Ω}, and concerns classes. NBG^{Ω} extends ZFC^{Ω} conservatively by distinguishing classes that are sets (namely, the classes that are a member[7] of some class) from proper classes (all others). Therefore, V is a proper class, also known as the universal class. NBG^{Ω} has the axiom of limitation of size, which asserts that all proper classes have an equal size: namely, Ω. The theory Morse–Kelley (Wang 1949) with the axiom of Global Choice (GC)[8] will be called MK, and also pertains to classes. MK extended with all the true formal axioms is MK^{Ω}. MK^{Ω} extends NBG^{Ω} by allowing for recombination (defining new entities via a formula in a comprehension schema) of classes instead of only sets. This makes MK^{Ω} a maximally expressive meta-theory of ZFC^{Ω} that can prove the existence of absolutely all classes, including that of the greatest class V.

according to P. Because a set is by definition always an element of some larger set, this makes P, paradoxically, itself formal. Indeed, there would be sets above x that could be recursively enumerated via the definition of x.

6. The problem with a formal theory F that has a formally defined number of formal axioms, is that it can easily be superseded by a slightly stronger formal theory F' that uses one or more extra formal axioms. This severely limits the value of F in a philosophical project.

7. While sets have elements, classes have members.

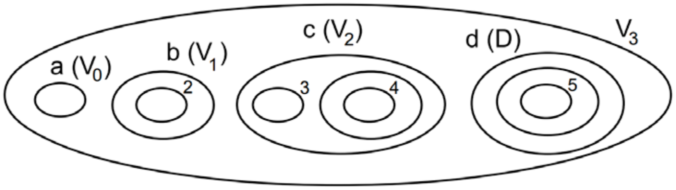
8. Without GC, comparability between proper classes sizes, and therefore the notion of a greatest entity, is lost.

Any axiom or definition that depends on the definition of the Absolute Infinite is not formal, but meta-formal. Therefore, the definitions of Ω, V, V^p, ZFC$^{\Omega}$, the axioms of NBG$^{\Omega}$ and MK$^{\Omega}$ that extend ZFC$^{\Omega}$, and several other definitions in this paper, are meta-formal. Nevertheless, meta-formal classes and class ordinals can be used in formal theories, which will consistently reinterpret their absolute nature as something less great than the true absolute nature. This will enable the claim that the existence of God can be proven in formal theories about metaphysical entities—if, at least, we adopt the inferior perspective of these formal theories. Only a meta-formal theory can truly prove God.

3.2. Foundational Mereological Definitions

For set-theoretic classes to be used as a basis for metaphysics, they need to have mereological properties, such as parthood, location, duplication relations, size and abundance (or multiplicity). Therefore, I will first introduce located duplicates. These are not classes, although they are intrinsic duplicates (henceforth 'duplicates') of some class.[9] Moreover, a located duplicate has a location that is defined by a chain of classes that are connected by 'is a member of' relations. The located duplicate is a duplicate of the smallest (first) class in the chain and has a location in all the classes in the chain. The largest class in which a located duplicate has a location is the locating class. Concrete examples of the definitions in this and the following paragraphs are given in Figure 1.

Figure 1: What is visualized here is a small class V_3 and the eleven located duplicates of which it is the locating class (V_3 has size ten). V_3 has four elements (or members): V_0, V_1, V_2 and D. These are duplicates of the located duplicates a, b, c and d respectively. The abundance of the empty set (V_0) in V_3 is five. V_0 and the five empty located duplicates a, 2, 3, 4 and 5 are all duplicates of each other.



Parthood will be defined as the transitive closure of the membership and the subclass relation, such that every class is a part of V. Parthood is reflexive, while proper parthood is irreflexive. The abundance of a class

9. Intrinsic duplicates have the same internal make-up, although they can have a different external environment.

A in a class B is equal to the number of located duplicates that are duplicates of A and that have B as their locating class. Every set has abundance one in itself and an absolutely infinite abundance in V. The size of a class A is equal to the number of located duplicates that are properly located in A. The empty set has size zero. The size of any proper class is equal to Ω. A relation R between two classes A and B (so A is R-related to B) holds iff a duplicate of A is R-related to a duplicate of B. Therefore, as a mnemonic, the phrase 'a duplicate of' may be added between brackets.

3.3. Foundational Metaphysical Definitions

3.3.1. The Central Axiom and the Theory $MK^{\Omega p}$

In order to prove the theological theorems in the Methodology section, the support of a central axiom is invoked that extends MK^{Ω}. It has a metaphysical and a set-theoretic formulation. V^p and V are the classes of, respectively, all non-absolute (or mundane) physical entities and all sets:

Metaphysical: The physical reality is rendered by set theory.
Set-theoretic: There exists a bijection[10] between V^p and V.

$$(\exists f{:}V^p \leftrightarrow V)$$

The central axiom is consistent with the axioms of ZFC, the Ω formal axioms that extend ZFC toward ZFC^{Ω}, the axioms of NBG^{Ω}, and the axioms of MK^{Ω} set theory. MK^{Ω} + the central axiom will be written as $MK^{\Omega p}$. The theological theorems in the first step of the proof can be proven in every sufficiently expressive formal fragment of the theory $MK^{\Omega p}$, even though only $MK^{\Omega p}$ itself can truly capture the intended meanings of the definitions.

3.3.2. Other Metaphysical Definitions

Proceeding via the definitions in the previous sections, we can construct metaphysical definitions for the entities that exist theoretically in $MK^{\Omega p}$. All these entities potentially exist in reality, and they are either concrete or abstract. Given the assumption of an ontologically parsimonious monism in the first step of the proof, all concrete entities are physical entities.

10. A bijection will be an all-to-all relation that works via one-to-one links. It will be a bidirectional injection, and will prove an equal size.

Every physical entity will be either mundane or transcendent. A mundane physical entity will be definable by a set. All its properties can be fully observed by observers who are themselves mundane physical entities. A physical entity that is too large or too complex to have a set model will be a transcendent physical entity. This means that it would require an absolutely infinite period of time for a mundane observer to observe each property of the entity at some point in time. All physical entities, mundane or transcendent, will be definable by a class.

Both worlds and the universe will be spacetimes. V^p, the union of all the mundane physical entities, will itself not be mundane but the transcendent physical universe, and will have every physical entity as a part. The universe will have an absolutely infinite size.

The remaining metaphysical and theological terms needed for proof of the theorems concerning V^p will be proposed in the subsection that proves these theorems. They include: conceivability, mereological and ontological greatness, and greatness (Theorem 1); causality, direct causation, and omnipotence (Theorem 2); omniscience and direct epistemic access (Theorem 3); omnipresence (Theorem 4); and existence in reality (Theorem 5).

3.4. Foundational Computer-Scientific Definitions

A machine will be a brain, a computer, a being, or a robot, and will be either physical or abstract. It produces an output from a given input. Several abstract models exist for machines: (1) a Turing machine (Minsky 1967), this being an ω-long tape with symbols and a moving read/write head that reads, (over)writes, or moves over the tape, or halts, according to a set of instructions, during ω time instants; (2) a cellular automaton (Wolfram 1984), where cells are made dead or alive according to the states of other cells and, again, a set of instructions; or (3) an artificial neural network (Sharma et al. 2012), consisting of a network of artificial neurons programmed in software that resembles the human brain. Machines that are Turing complete (a requirement that is easily fulfilled) can, given enough (transfinite) time and memory resources, solve any computation problem and simulate any world.

Before producing the next bit of information in the output, a Turing complete machine can consult the whole input. This implies that causal consequences of an elementary section of the memory contents of a machine are non-local: they can travel without any delay through the rest of the memory contents. Consequently, in contrast to Lewis' (1986) modal realism, and in spite of our limited empirical observations, there is no causal isolation in a set-theoretic realism.

Machines are either absolute or ordinal. An ordinal machine (Koepke and Seyfferth 2009) is one of the following three machines: a hypercomputer with transfinite uncountable resources, a Turing machine with countable transfinite resources, or a computer with finite resources. Ordinal machines are, therefore, the ordinal extensions of computers with finite resources. Their available time and memory (tape length, cell matrix, number of neurons, etc.) have ordinal values. Every mundane machine is a physical ordinal machine and is a part of a computationally stronger mundane machine, which has more available time and memory. The Absolute Machine has absolutely infinite available time and memory. The whole physical reality is both observed and computed by the Turing complete Absolute Machine V^P. Even though V^P makes multiplicative computations that include itself as input, it does not output entities larger than V^P, but at most ones identical to V^P.

4. METHODOLOGY

Because our proof of God depends on the central axiom, the latter will first be provided with a philosophical justification. Then the proof will proceed in two steps: in the first of these, it will be proven that a physical God with omni-attributes is the greatest conceivable entity, and in the second, the criterion of ontological greatness will be altered so that a non-physical, concrete God or an abstract God is proven to be the greatest conceivable entity.

4.1. The Central Axiom: Philosophical Justification

The central axiom states: There exists a bijection between V^P, the class of all mundane physical entities, and V, the class of all sets in MK^Ω set theory. The philosophical justification of the central axiom is that both physical entities and classes contain information.[11]

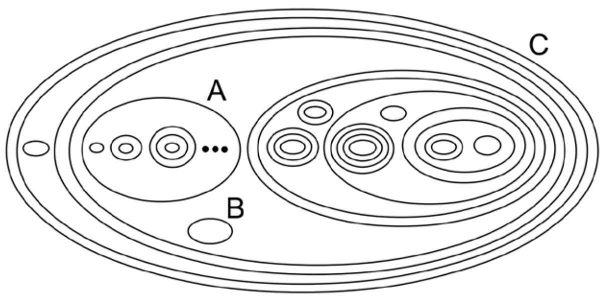
A sufficiently large class contains information that can be derived through a statistical analysis of the parthood relations among the parts of the class. Just like for strings of symbols, the number and size of recurring patterns determine the amount of information.[12] In Shannon's (2001) information theory, the average amount of information in a string of symbols is called 'entropy.' More disordered (entropic) strings contain more

11. In fact, there are at least four kinds of abstract entities that have a bijective correspondence with information entities: sets, ordinal numbers, ordinal machines, and ordinal strings of symbols.

12. The conversion to strings of symbols can easily be made via the notation with curly brackets of pure sets. An example is {{},{{}}}.

information, whereas more ordered (redundant) strings are easier to analyze. Only classes that are highly ordered or very easy to define can be said to contain little or no information. Three examples are given in Figure 2.

Figure 2: The three classes *A*, *B*, and *C* have various degrees of entropy, or disorder. Class *A* has an infinite size ω, the smallest infinity. Yet it has so much order that, just like the empty class *B*, it contains no information. Class *C*, which has size ω+24, contains roughly 24 bits of information.



The key argument for why abstract entities suffice to bring about physical entities is that observers simulated by an abstract machine cannot conduct any experiment to find out whether they are simulated by an abstract machine or a physical machine with the same information contents. Abstract observers will, therefore, firmly believe they live in a physical world.

4.2. Step 1: A Physical God with Omni-Attributes

Having defended the central axiom and introduced the foundational definitions pertaining to set theory, mereology, metaphysics and computer science in the theory $MK^{\Omega p}$, we are now in possession of the starting material to construct some further definitions and proofs about V^p in this theory. This V^p will be shown to be our preliminary physical God in Theorem 1.

4.2.1. Theorem 1: The Unique Maximal Greatness of V^p

Theorem 1 (V^p is uniquely the greatest of absolutely all conceivable entities) follows from definitions of conceivability, mereological greatness and greatness, and from a criterion of ontological greatness.

We can therefore consider the following definitions and criterion: an entity *A* is conceivable iff it exists theoretically according to a formal or meta-formal theory. An entity *A* is greater than an entity *B* iff either (1) *A* is ontologically greater than *B*, or (nonexclusively) (2) *A* is mereologically greater than *B*, and *B* is not ontologically greater than *A*. An entity *A* is mereologically greater than an entity *B* iff *B* is a proper part of *A*.

Criterion: if an entity *A* is physical and an entity *B* is not physical, then *A* is ontologically greater than *B*.[13]

Proof. V^p is conceivable, given that it exists theoretically according to $MK^{\Omega p}$. Moreover, absolutely all other conceivable entities are less great than V^p, because absolutely all other entities that are ontologically equally great compared to V^p, such as worlds, are mereologically less great than V^p. Therefore, V^p is uniquely the greatest of absolutely all conceivable entities. □

Because V^p is uniquely the greatest of absolutely all conceivable entities, it is equal to Anselm's God.

4.2.2. Theorem 2: The Omnipotence of V^p

Theorem 2 (V^p is omnipotent) follows from definitions of a Turing complete Absolute Machine, the 'causes' relation, direct causation, and omnipotence.

We can consider the following definitions: An entity *A* causes an entity *B* iff a machine or process uses (a duplicate of) *A* as input to bring about, simulate, or output (a duplicate of) *B* within some spacetime. An entity *A* directly causes an entity *B* iff *A* causes *B*, and no distinct intermediate entity *C* is in the causal chain between *A* and *B*. An entity is omnipotent iff it causes absolutely every physical entity directly.

Proof. As a physical, Turing complete Absolute Machine, V^p uses itself as input to bring about absolutely every physical entity. Therefore, V^p causes absolutely every physical entity. Because absolutely every physical entity is a part of V^p, V^p causes absolutely every physical entity directly. Therefore, V^p is omnipotent. □

4.2.3. Theorem 3: The Omniscience of V^p

Theorem 3 (V^p is omniscient) follows from a definition of omniscience and direct epistemic access. We can consider the following definitions: An entity is omniscient iff it has direct epistemic access to absolutely every conceivable entity. Epistemic access is direct iff it is without inference, mediation, or delay.

Proof. Because V^p is a Turing complete machine, it has direct epistemic access to every entity in its memory. Moreover, every entity that is conceivable via $MK^{\Omega p}$ is in the memory of the physical Absolute Machine V^p. That is absolutely every conceivable entity. Consequently, V^p has direct epistemic access to absolutely every conceivable entity. Therefore, V^p is omniscient. □

13. This criterion will be altered in Step 2 of the proof.

4.2.4. Theorem 4: The Omnipresence of V^p

Theorem 4 (V^p is omnipresent) follows from a definition of omnipresence. We can therefore consider the following definition: An entity A is omnipresent iff A is present at absolutely every physical location in the sense that it is able to act upon and be aware of absolutely every physical event wherever it occurs.

Proof. Given that absolutely every physical entity is a part of V^p and computed by V^p, V^p is able to act upon and be aware of absolutely every physical event wherever it occurs. Therefore, V^p is omnipresent. □

4.2.5 Theorem 5: The Existence in Reality of V^p

Theorem 5 (V^p exists in reality) follows from a definition of existence in reality of entities and from a corollary of Theorem 2 (V^p causes absolutely every physical entity). We can consider the definition that a physical entity exists in reality iff (1) it potentially exists in reality, and (2) it is causally related to our actually observed world.

Proof. All the entities that exist theoretically according to $MK^{\Omega p}$ potentially exist in reality. Since V^p exists theoretically according to $MK^{\Omega p}$, it potentially exists in reality. According to a corollary of Theorem 2, V^p causes absolutely every physical entity, including our actually observed world. In this way, V^p fulfills all the requirements for existence in reality. Therefore, V^p exists in reality. □

Given the lack of any causal separations in V^p, absolutely every possible physical entity is required as something that exists in reality, including V^p itself. Together with Theorem 1 and Anselm's definition of God, it then follows that God exists in reality.

4.3. Step 2: A Non-Physical God

In a longstanding tradition, classical natural theology developed the following hierarchy of ontological greatness, from least great to greatest (Leftow 2012):

1. Physical (causal agency, spatiotemporal, contingent, perishable).
2. Abstract (causally inert, non-spatiotemporal, necessary, unchanging).
3. Concrete non-physical (causal agency, personal, imperishable).

From the point of view of the central axiom, this hierarchy is no longer sustained in the same way, because exactly the same entities can be found for physical and for abstract entities. For example, the output of an abstract Turing machine is indistinguishable from that of a physical Turing machine.

Both have the same causal powers and create entities with the same properties. Nevertheless, there remains a difference in perception between the categories. Physical entities are observed by us as very large, but finite, outputs of an Absolute Machine. Abstract entities can be mentally constructed by us, starting from the very smallest units, even though they exist independently of our minds and explain the physical entities. Classically, concrete non-physical entities inherit the best properties of the physical and the abstract entities. For at least categories (1) and (2), the same two levels can be distinguished:

1. Divine: transcendent, absolute, meta-formal, and proper--class-like.
2. Worldly: mundane, exceedable, formal, and set-like.

This analysis makes it hard to decide whether the ontological categories are really different, and also how they relate with respect to ontological greatness. Therefore, I shall adopt the ontology of classical natural theology, with non-physical concrete entities, physical entities and abstract entities, without proposing any hierarchy as to superiority.

If we add a non-physical, concrete entity that potentially exists in reality to what exists theoretically,[14] then we are faced with the question of what is ontologically greatest in Theorem 1: is it the non-physical, concrete entity, physical entities, or abstract entities? According to theorems 2, 3, 4 and 5 in Step 1, there is an entity in reality that is omnipotent, omniscient and omnipresent (namely V^p). With that, we have the following parameterizable proof for either a concrete, non-physical or an abstract God that modifies Theorem 1 and the criterion of ontological greatness:

1. God is uniquely the greatest of absolutely all conceivable entities.
2. Non-physical (either concrete or abstract) entities are ontologically the greatest.
3. If there is an entity in reality with omni-attributes, then God exists in reality and has these omni-attributes.
4. V^p, the physical universe, is an entity in reality that is omnipotent, omniscient and omnipresent.
5. Therefore, God is non-physical (either concrete or abstract), exists in reality, and is omnipotent, omniscient and omnipresent.

Depending on what is ontologically the greatest, we get three possible outcomes: a non-physical, concrete God (the God of classical natural theism), a physical God (V^p), or an abstract God (V).

14. This results in a new language $MK^{\Omega c}$ that has three ontological categories.

5. Findings

The main finding of this paper is that God and His attributes can be defined and proven using modern formalisms. Beyond this, three additional conclusions will be highlighted in this section: (1) that the central axiom exhibits great explanatory power, (2) that God and the Absolute Infinite can be consistently defined, and (3) that in spite of modal realism, the worlds in V^p are not isolated.

5.1. The Explanatory Power of the Central Axiom

Apart from sustaining a proof of God, the central axiom explains why there are physical entities at all. Even if there were no physical entities in reality, abstract entities would still exist. We cannot imagine a reality in which, for example, the thirteenth natural number would not be a prime number. Moreover, in contrast to the standard view (Juvshik 2018), abstract entities have causal relations. This is especially apparent for abstract ordinal machines, whose computations and simulations bring about (or cause) abstract worlds. The proposal of the central axiom is essentially that the abstract worlds that are brought about in the simulations of abstract machines explain the physical worlds, and that V explains V^p.

5.2. The Consistency of God

Cantor, who introduced Ω, thought of it as an inconsistent multiplicity (Jané 1995), defining it as a set ordinal that exceeds every set ordinal. This may be one of the reasons why Ω has not been popular as a concept among set theorists, who want to avoid inconsistencies in their formal theories. For theologians, an inconsistent God does not look attractive either. My view here is (1) that Ω is a proper class ordinal (and a proper class cardinal), and (2) that the true conception of Ω, and, as a consequence, the true conception of God, cannot be formally defined. This fits much better with our intuitions about God than an inconsistent God.

5.3. The Invalidity of Modal Realism

Lewis' (1986) claim that worlds are causally and spatiotemporally isolated is a challenge to the core of the set-theoretic realism proposed in this paper. In order to provide more insight into this conflict, we could establish a proof from contradiction by assuming that V^p is the universe of all the causally isolated worlds and that causal activity only happens within those worlds and not within V^p as a whole. We thereby assume that each world W exists as a causally isolated version, such that it only does so as a member (rather than a part) of V^p, with abundance one. However, taking into account all the duplicates of W (that stem from recombination of W with other worlds), the world W exists with an absolutely infinite abundance in V^p as a causally interactive part. Assuming that the observation of a world in V^p is random and based on equal probabilities, it follows that the probability that we observe anything causally isolated from V^p will be absolutely infinitely small. Even though isolated worlds may remain technically 'possible,' they are non-actual with certainty. Likewise, it may be technically possible that a randomly selected real number is an integer; however, the probability that this will happen is zero.

6. Discussion

This section contains a discussion of (1) differences from classical natural theology, (2) the provability and conceivability of God, (3) God's omni-attributes, (4) which set-theoretic variant to choose, (5) modal realism and set-theoretic realism as theological projects, and (6) the compatibility of set-theoretic realism with the findings of the empirical sciences.

6.1. Differences from Classical Natural Theology

Classical natural theology often injects deep complexity into metaphysics because it introduces concepts that both overlap with and transcend ordinary metaphysical categories (Swinburne 1993): divine foreknowledge and free will, divine providence and determinism, omnipotence and logical coherence, omnibenevolence and evil, divine action and causation, simplicity and the possession of components, the soul and consciousness, eternity and time, and so on. This complexity often stems from the classical assumption that God is clearly distinct from the physical universe and the sentient beings in it, and that He belongs to a different ontological category.

The assumption of a set-theoretic metaphysics significantly simplifies the complex relation between God and physical reality, because God is a limit case of physical reality with respect to the Absolute Infinite. In this case, God's nature is continuous with the nature of arbitrarily complex, physical agents with the computational power of transfinite ordinal machines. For example, God's omnibenevolence might be derivable from the collective benevolence of such ordinal machines.

One of the consequences of this simplification is that the proposed theology is most easily interpreted as a form of pantheism or panentheism, especially for those who argue that physical or abstract entities are ontologically the greatest. Also the three proposed omni-attributes are simplified. For example, while omniscience in classical natural theology does not imply that God undergoes our pain, the proposal that God is an Absolute

Machine that computes our pain makes this interpretation more difficult. Nevertheless, it can be argued that God does not suffer by subjectively participating in our pain, because God's brain is absolutely infinitely larger than our brains. Therefore, God's suffering can be ignored.

6.2. Provability and Conceivability

As mentioned in Section 3.3.1, the theological theorems in the first step of the proof can be proven in every sufficiently expressive formal fragment of the theory $MK^{\Omega p}$, such as, for example, MK set theory extended with the definitions introduced in this paper (resulting in MK^{p}). Nevertheless, from the meta-formal perspective of $MK^{\Omega p}$, the absolute entities that are proven to exist via the formal MK^{p} are less great than those that can be proven by $MK^{\Omega p}$ itself. MK^{p} holds itself capable of conceiving of God and other absolute concepts, while $MK^{\Omega p}$ recognizes this formal conception as something that is less great as compared to its own true conception of God and these concepts. In other words, while both proving and conceiving God can be done in formal theories, it is only if we admit that the act of conceiving something can be arbitrarily theoretical that we can use the meta-formal theory $MK^{\Omega p}$ to conceive of God as seen from the absolutely greatest perspective. After all, formal theories such as MK^{p} also make use of the infinitary recursive enumeration of a Turing machine to conceive of their God.

6.3. God's Omni-Attributes

Three omni-attributes were defined via modern formalisms: omnipotence, omniscience, and omnipresence. A fourth omni-attribute, omnibenevolence, was found to be too difficult to define and prove in this way. Nevertheless, Blondé (2015) shows that V^{p} is benevolent as a result of the fact that evil agents often behave benevolently in order to appear benevolent, while benevolent agents never behave malevolently for any reason. Other attributes of God, such as eternity, transcendence and immutability, might be easier to define and prove, but are beyond the scope of this paper.

6.3.1 Omnipotence

A question that has often been investigated is whether God can cause evil (Mackie 1955). According to the definition of omnipotence, God causes evil, given that every evil world exists and God causes every world. A related question is whether a universe that is mereologically maximal in size, but that includes evil worlds, can be as great as or greater than a universe that does not include evil worlds (Kraay 2017). I argue that this is indeed the case, because evil worlds can be brought into a state that is equivalent to non-existence with respect to axiology.

In a mereologically maximally great universe, worlds do not all have equal abundances. For example, non-reproducing worlds and worlds that are not favored by powerful ordinal machines will be outnumbered by worlds that reproduce or are reproduced abundantly. In particular, God can reproduce benevolent worlds with an absolutely infinitely greater abundance than evil worlds. In this case, the evil worlds are with certainty not observed, which reduces them effectively to non-existent worlds.

6.3.2 Omniscience

Simoni (1997: 2) investigated the problem of radical particularity: how can a universal, boundless being know what is experienced by radically finite beings? V^p knows the answer to every mathematical problem that requires transfinitely long computations, such as the halting problem (Lucas 2021). It also knows every mundane experiential fact from its own experience (because every mundane machine—including, for example, every neural network—is a part of V^p), and it knows every indexical (e.g. temporal) fact relative to a world. Moreover, because every recombination of V^p with itself results in V^p, (a duplicate of) every formal fact (including every finite fact) is taking place at some location at each temporal instant of V^p's existence. Therefore, God knows everything during every instant of time, including our human experiences.

6.3.3 Omnipresence

In analyzing omnipresence, Hudson (2009) distinguishes several mereological relations between physical entities and locations, such as being entirely located at, being wholly located at, and being partly located at. For each physical entity, these three relations hold with respect to the location V^p.

Hartshorne (1941), in arguing for a non-physical God, proposed the analogy that God is to the world like a mind is to its body. Therefore, God has immediate knowledge and direct power over every part of the universe. Indeed, as a Turing complete Absolute Machine, V^p has immediate and direct read and write access to its entire memory contents (which is V^p itself).

6.4 Which Set-Theoretic Variant to Choose?

One could argue that the set-theoretic theory ZFC, which is the canonical theory in mathematics, cannot prove the existence of a greatest entity. However, this only shows that ZFC can be dismissed as being too weak to prove the existence of a series of meaningful meta-concepts, such as the universal class. Seeking refuge in this argument comes down to admitting that every arbitrarily godlike entity provably exists in the physical universe, but that the physical universe itself does not provably exist. In natural language (such as English), we have all the meaningful concepts and meta-concepts at our disposal. A correct translation into set theory can therefore be arrived at using the theory MK, which is a sufficiently strong meta-theory of ZFC.

6.5. Proving God via Modal Realism

Modern formal attempts to prove the existence of God, such as Plantinga's (1978) and Gödel's ([1941] 1995) ontological arguments, have made use of modal logic. According to modal logic, things exist possibly when they exist in some possible world, and necessarily when they exist in every possible world. Gödel defines God as having all and only the positive properties in a given possible world, and claims that existence is a positive property. He then uses a strict system of axioms and definitions to show that such a God exists in every possible world. Therefore, God exists necessarily. Plantinga posits that if God exists possibly, then God exists necessarily, and also that it is possible that God exists. Therefore, again, God exists necessarily.

What remains unclear is whether Gödel and Plantinga have shown the existence of one God only, or one God for each possible world. We get many gods if all the possible worlds are causally and spatiotemporally isolated and just as real as the actual world (our world), as advocated by Lewis. Moreover, if some possible worlds have few positive properties apart from existence, Gödel's God is not necessarily a great being. Given these problems with modal realism, the set-theoretic metaphysics becomes an important extra option for the theist who wants to prove a unique, maximally great God.

6.6. Compatibility with the Empirical Sciences

The most significant research gap in this paper pertains to explaining why the observable universe is not extremely complex and benevolent, and why the empirical sciences are so successful. As an explanation, I refer to the present author's account of evolutionary conservation in cosmological natural selection (Blondé 2016, 2019). According to this theory, biological organisms with more than three spatial dimensions require vastly more time to develop Darwinistically. Because of this, they emerge in a world in which three-dimensional organisms like us are already present. Some of them will therefore extend, reuse, or build on the complexity that we create, such that they become evolutionarily dependent on us. This implies that these 3D-extenders have to operate with the greatest respect for the evolutionarily conserved reproduction plan of our observable universe, in order to maintain and reproduce themselves. Assuming that the knowledge gathered by our empirical sciences has become evolutionarily conserved long before the advent of higher-dimensional organisms, it means that the latter have to be very subtle when they interfere. On the other hand, they will want to reproduce our observable universe as much as possible in order to reproduce themselves. Consequently, they will simulate our observable universe in a way that does not alter the knowledge gathered by our empirical sciences, though with less spatial requirements. This logic can repeat itself for absolutely infinitely many spatial dimensions, via 4D-extenders, 5D-extenders, etc.

This theory is compatible with the requirement of Blondé and Jansen (2021) that V^p consists primarily of conscious, intelligent matter (brain matter or CPU matter) if it is to solve the 'hard problem' of consciousness (Chalmers 2017). Because complexity in lower dimensions is always more efficiently simulated in higher ones, the density of conscious, intelligent matter will increase as we pass over, at the limit, to absolutely infinitely many spatial dimensions. The ultimate reality, therefore, will consist for 100% of conscious, intelligent matter, which we can call 'God's brain.' However, God's brain creates experiences of an evolutionarily conserved external world that is located in an absolutely infinitely small fraction of V^p.

7. Conclusions

The deductive proofs of five theological theorems on the basis of MK set theory extended with absolutely infinitely many axioms, a central axiom, and a list of definitions show that it is possible to define and prove the existence of God, and His attributes, via modern formalisms, if at least the central axiom and the definitions are accepted. Three of God's omni-attributes—omnipotence, omniscience, and omnipresence—appear to be translatable into a merger of set theory, mereology, computer science, metaphysics and natural theology. As regards the proof of these attributes, God is, in the first step, identified with a physical analogue of the von Neumann universe of sets V: namely, the universe of all mundane physical entities V^p. In the second step, we have found it possible to reuse the preliminary proof of a physical God as a proof of a non-physical, concrete God, or an abstract God.

Some other important findings are that the central axiom explains both God and physical reality, that God and the Absolute Infinite can be consistently defined, and that modal realism with its isolated worlds is invalid. The philosophical justification of the central axiom is that mathematical truths and abstract entities exist independently of physical reality. This includes ordinal machines and the Absolute Machine (which is God).

This approach to translating theological terminology into modern formalisms has also revealed some limitations. First, the translation provides no insight into how the existence of absolutely infinitely many physical entities can be reconciled with the observations of the empirical sciences. The solution probably has to be found in other paradigms, such as evolutionary conservation in cosmological natural selection. Second, not all theological terms are suited to being translated into modern formalisms. Omnibenevolence is one example.

In conclusion, I recommend giving up on Lewis' isolated worlds in modal realism. They may exist, but the probability that our observable universe is part of one is zero. Instead, theists would do better by using Cantor's paradise of a unifying set-theoretic realism to prove God.

References

Blondé, Ward, and Ludger Jansen. 2021. "Proving God without Dualism: Improving the Swinburne-Moreland Argument from Consciousness." *Metaphysica* 22 (1): 75–87. https://doi.org/10.1515/mp-2020-0035.

Blondé, Ward. 2015. "An Evolutionary Argument for a Self-Explanatory, Benevolent Metaphysics." *Symposion* 2 (2): 143–66. https://doi.org/10.5840/symposion2015228.

—. 2016. "Can an Eternal Life Start from the Minimal Fine-Tuning for Intelligence?" *Philosophy and Cosmology* 17: 26–38.

—. 2019. "EMAAN: An Evolutionary Multiverse Argument against Naturalism." *Symposion* 6 (2): 113–28. https://doi.org/10.5840/symposion2019629.

Cantor, Georg. 1883. "Über unendliche, lineare Punktmannichfaltigkeiten." *Mathematische Annalen* 21 (4): 545–91.

Chalmers, David J. 2017. "The Hard Problem of Consciousness." In *The Blackwell Companion to Consciousness*, edited by Susan Schneider and Max Velmans, 32–42. 2nd ed. Malden, MA: Wiley-Blackwell.

Forgie, J. William. 2000. "Kant and Frege: Existence as a Second-Level Property." *Kant-Studien* 91 (2): 165–77. https://doi.org/10.1515/kant.2000.91.2.165.

Geach, Peter. 1956. "Good and Evil." Analysis 17 (2): 33–42. 10.1093/analys/17.2.33.

Gödel, Kurt. (1941) 1995. "Ontological Proof." In *Collected Works*, vol. 3, *Unpublished Essays and Lectures*, edited by Solomon Feferman, John W. Dawson Jr., Warren Goldfarb, Charles Parsons, and Robert N. Solovay, 403–4. Oxford: Oxford University Press.

Gödel, Kurt. 1931. "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I." *Monatshefte für Mathematik und Physik* 38 (1): 173–98.

Hamkins, Joel David. 2012. "The Set-Theoretic Multiverse." *Review of Symbolic Logic* 5 (3): 416–49. https://doi.org/10.1017/S1755020311000359.

Hartshorne, Charles. 1941. *Man's Vision of God and the Logic of Theism.* Chicago: Willett, Clark & Co.

Hudson, Hud. 2009. "Omnipresence." In *The Oxford Handbook of Philosophical Theology*, edited by Thomas P. Flint and Michael C. Rea, 199–216. Oxford: Oxford University Press.

Jané, Ignacio. 1995. "The Role of the Absolute Infinite in Cantor's Conception of Set." *Erkenntnis* 42 (3): 375–402. https://doi.org/10.1007/BF01129011.

Juvshik, Tim. 2018. "Abstract Objects, Causal Efficacy, and Causal Exclusion." *Erkenntnis* 83 (4): 805–27. https://doi.org/10.1007/s10670-017-9915-1.

Koepke, Peter, and Benjamin Seyfferth. 2009. "Ordinal Machines and Admissible Recursion Theory." *Annals of Pure and Applied Logic* 160 (3): 310–8. https://doi.org/10.1016/j.apal.2009.01.005.

Kraay, Klaas J. 2017. "Invitation to the Axiology of Theism." In *Does God Matter? Essays on the Axiological Consequences of Theism*, edited by Klaas J. Kraay, 1–35. New York: Routledge.

Leftow, Brian. 2012. *God and Necessity.* Oxford: Oxford University Press.

Lewis, David. 1986. *On the Plurality of Worlds.* Oxford: Blackwell.

Lucas, Salvador. 2021. "The Origins of the Halting Problem." *Journal of Logical and Algebraic Methods in Programming* 121:100687. https://doi.org/10.1016/j.jlamp.2021.100687.

Mackie, John L. 1955. "Evil and Omnipotence." *Mind* 64 (254): 200–12. https://doi.org/10.1093/mind/LXIV.254.200.

Marenbon, John. 2015. "Anselm: Proslogion." In *Central Works of Philosophy.* Vol. 1, *Ancient and Medieval*, edited by John Shand, 45–67. Durham: Acumen Publishing Limited.

Minsky, Marvin L. 1967. *Computation: Finite and Infinite Machines.* Englewood Cliffs, NJ: Prentice-Hall.

Neumann, John von. 1928. "Die Axiomatisierung der Mengenlehre." *Mathematische Zeitschrift* 27 (1): 669–752.

Oppy, Graham, ed. 2018. *Ontological Arguments.* Cambridge: Cambridge University Press.

Plantinga, Alvin. 1978. *The Nature of Necessity.* Oxford: Clarendon Press.

Schmidhuber, Jürgen. 2006. "A Computer Scientist's View of Life, the Universe, and Everything." In *Foundations of Computer Science: Potential—Theory—Cognition*, edited by Christian Freksa, Matthias Jantzen, and Rüdiger Valk, 201–8. Berlin: Springer.

Shannon, Claude E. 2001. "A Mathematical Theory of Communication." *ACM SIGMOBILE Mobile Computing and Communications Review* 5 (1): 3–55. https://doi.org/10.1145/584091.584093.

Sharma, Vidushi, Sachin Rai, and Anurag Dev. 2012. "A Comprehensive Study of Artificial Neural Networks." *International Journal of Advanced Research in Computer Science and Software Engineering* 2 (10): 278–84.

Simoni, Henry. 1997. "Omniscience and the Problem of Radical Particularity: Does God Know How to Ride a Bike?" *International Journal for Philosophy of Religion* 42 (1): 1–22. https://doi.org/10.1023/A:1002922511041.

Smolin, Lee. 1992. "Did the Universe Evolve?" *Classical and Quantum Gravity* 9 (1): 173–91.

Steinhart, Eric. 2010. "Theological Implications of the Simulation Argument." *Ars Disputandi* 10 (1): 23–37. https://doi.org/10.1080/15665399.2010.10820012.

Swinburne, Richard. 1993. *The Coherence of Theism.* Rev. ed. Oxford: Clarendon Press.

Tegmark, Max. 2008. "The Mathematical Universe." *Foundations of Physics* 38: 101–50. https://doi.org/10.1007/s10701-007-9186-9.

Wang, Hao. 1949. "On Zermelo's and von Neumann's Axioms for Set Theory." *Proceedings of the National Academy of Sciences* 35 (3): 150–5. https://doi.org/10.1073/pnas.35.3.150.

Wolfram, Stephen. 1984. "Cellular Automata as Models of Complexity." *Nature* 311 (5985): 419–24. https://doi.org/10.1038/311419a0.

Zermelo, Ernst. 1908. "Untersuchungen über die Grundlagen der Mengenlehre I." *Mathematische Annalen* 65 (2): 261–81. https://doi.org/10.1007/BF01449999.