Elucidating the Role of Truth-Expressions
Some Wittgensteinian Grammatical Investigations

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Abstract The aim of this text is to elucidate certain aspects of the use of expressions such as "is true" and "it is true that" (henceforth "truth-expressions") and, through this, some features of the concept of truth. It focuses on addressing the question of whether truth-expressions play the role of a predicate or an operator. The investigations pursued are intended to be grammatical—in Wittgenstein's sense of the term. I begin with a short presentation of a widely held view about the role played by truth-expressions. I then contrast the Wittgensteinian conception of grammar with that of linguistics. I sketch Frege's, Wittgenstein's, Prior's and Brandom's central ideas regarding the issue under consideration. As a further step, I investigate the role of truth-expressions by examining several sentences in which they occur, and discuss objections to the proposed analysis. On my approach, truth expressions play the role of a predicate only when applied to sentences, and in all other cases function as operators. One advantage of such a position is that it enables a dissolution of the problem of truth-bearers: where truth-expressions are operators, the issue simply does not arise, and where they are predicates, it is sentences that are the truth-bearers.

Keywords grammatical investigations; truth; truth predicate; truth operator; Wittgenstein, Ludwig

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1. The Basic Problem: What Role Do Truth-Expressions Play in our Language?

Truth can be perceived as something both utterly straightforward and highly enigmatic. Until one actually poses, like Pilate, the question "What is truth?," its nature does not seem in any way problematic. After all, any of us can employ sentences constructed along the lines of "It is true that it is so," "Everything she said is true," "Such and such a statement is true," etc., and comprehend them perfectly. However, as soon as the question is asked, it turns out that it is by no means easily answered, and that every answer offered so far can be considered in some way or other controversial. Thus, the concept of truth seems, on the one hand, easy to grasp, but on the other, hard to explain.

Nevertheless, it may well be that the problem is not generated by the concept itself, but rather by the manner in which one asks about the nature of truth, and the fact that one is ready and willing to accept as satisfactory only an answer of a specific kind. According to a certain widely adopted approach, the question "What is truth?" should be understood as asking whether there is a property to which the word "truth" refers and, if there is one, what the nature of this property is. The latter also seems to assume that any satisfactory answer to this question ought to take the form of a theory that would somehow explain the use of expressions containing the word "truth" and its derivatives (henceforth "truth-expressions").

In this text, I wish to advocate approaching the question of the nature of truth from another perspective: namely, the Wittgensteinian one. According to Wittgenstein, philosophy should not seek to advance theories (2009, § 109), but should aim instead at removing the conceptual confusions that generate philosophical problems. If one embraces such a perspective, then in order to answer the question "What is truth?" one need only give an adequate description of the use of truth-expressions. This does not make it an easy task, because giving such a description requires, amongst other things, taking into account both similarities and differences between the use of truth-expressions and that of other expressions. So, to answer the question one should draw such analogies as will furnish insights into how truth-expressions are employed in our language, but without misleadingly suggesting that these expressions function in a similar way to others. In some sense, then, my considerations regarding the essence of truth can be viewed as proceeding under the guidance of Wittgenstein's remark that "*Essence* is expressed in grammar" (2009, § 371). Here, it should be emphasized that the Wittgensteinian notion of grammar differs from that employed in both traditional and contemporary linguistics.

Before I start to discuss the question of the nature of truth in detail, I intend to make some clarificatory remarks about the character of this text and its subject matter. Firstly, I should say that I will not be presenting a comprehensive interpretation of Wittgenstein's remarks on truth, even though I will be appealing to some of them, as my goal is to approach the question of truth from a Wittgensteinian perspective rather than developing an exegesis of his own statements on that topic. Secondly, in this text, I shall not discuss all possible uses of truth-expressions, as doing so would probably call for a book-length study. Moreover, the text focuses, basically, on just one way of using these expressions. I try to point out that in many cases—contrary to appearances—truth-expressions play the role of an operator, not a predicate, and that realizing this fact enables one to avoid various philosophical problems generated by a mistaken descriptive account of the grammar of these expressions. (I should also add that I am not going to discuss the Liar Paradox at all, even though dealing with the latter is considered by many philosophers to be one of the most important tasks connected with the analysis of the concept of truth. The reason is simple: it would require an analysis extending at a minimum to several pages. It should just be noted that the approach to truth proposed here does potentially have consequences for the (dis)solution of that paradox [see: Grover 2005; Mulligan 2010].) Thirdly, I wish to note that although the following investigations into the nature of truth should be treated as grammatical in the Wittgensteinian sense of that word, I am only going to discuss Wittgenstein's notion of grammar and conception of grammatical investigations in the broadest terms. Fourthly, I would like to strongly emphasize that the arguments I have presented in favour of the approach to truth-expressions proposed in the text should not be taken as proof that the grammar (in the Wittgensteinian sense of the term) of truth-expressions is so-and-so. Rather, my aim is to convince the reader that the linguistic phenomena under consideration can be viewed in the manner proposed here (see Diamond 2004, 211–3; Kuusela 2008, 262). The presentation of this possibility is to show that certain common ways of thinking about the functioning of truth-expressions are not as obvious and without alternatives as they seem. Moreover, showing that an approach to the use of truth-expressions different from the standard approach is possible may lead to liberation from some philosophical problems—problems, the main source of which is that we cling to an analogy, that we are held captive by a certain picture (see Wittgenstein 2009, § 115). (As the commentators of Wittgenstein's writings point out, when we consider a philosophical problem, trying to understand a phenomenon by an analogy, substituting one

analogy for another can help dissolve the problem, because the source of the problem may have been a misleading analogy [see Diamond 2004, 211–2; Kuusela 2008, 88].) In this text, I try to show that by adopting a different approach to truth-expressions than the standard approach, the problem of the so-called truth-bearers can be dissolved.

The structure of the remainder of the article is as follows. The second part contains a presentation of an issue that is key to understanding the main topic of this text, this being the question of whether truth-expressions play the role of predicates or operators. In the third section, I then briefly discuss Wittgenstein's conception of grammatical investigations. In the fourth, I recall some ideas of four philosophers (Frege, Wittgenstein, Prior and Brandom) whose approaches to the question of truth are in some respects very close to that put forward here. In the fifth part, some examples of different types of sentences containing truth-expressions are presented. The sixth section is devoted to clarifying the use of truth-expressions in these sample sentences, and to discussing various possible ways of classifying such sentences. In the seventh and eighth parts, some objections pertaining to the adequacy of the type of approach to the use of truth-expressions being advocated by me are discussed. In the ninth part of the text, I juxtapose the employment of truth-expressions as a predicate with their use as an operator. Finally, in the last section, I show how elucidations of the use of truth-expressions can be used to dissolve the problem of truth-bearers.

2. The Fundamental Controversy: Do Truth-Expressions Play the Role of Predicates or Operators?

The view that truth-expressions—that is, expressions such as "is true" and "the truth is"—play the role of a predicate that refers to the property of being true is a commonly held one (see Dummett 2000; Horwich 1990; Künne 2003; Russell 2001; Soames 2003). Such a position may at first glance seem not only right, but also uncontroversial. According to both traditional grammar and several interpretations put forward in the context of contemporary linguistic theories, the expression "is true," when it occurs in sentences such as "The Pythagorean theorem is true," "That the climate is getting warmer is true," or "It is true that Wittgenstein appreciated Frege very much," functions as a predicate. Moreover, many philosophers and logicians (e.g. Davidson 2005; Dummett 2000; Horwich 1990; Russell 2001; Soames 1999; Tarski 1944) assume that this predicate refers to the property of being true. If one accepts this view, one will be inclined to pose the following question: to what kind of objects should the property of being true be ascribed? This is a question about what have come to be known as "truth-bearers," to

which many different answers have been put forward. Some philosophers claim that propositions are the real truth-bearers, others that they are furnished by sentences (or threefold items consisting of sentences, persons and moments of time), and still others that they are statements.

But are the disputes surrounding truth-bearers and the nature of the property of being true really instances of a genuine controversy? Such a seemingly outrageous question can be raised, providing one is prepared to doubt the necessity of interpreting sentences such as "The Pythagorean theorem is true," "That the climate is getting warmer is true," or "It is true that Wittgenstein appreciated Frege very much," as being subject-predicate ones. According to traditional grammar, in the first sentence the expression "the Pythagorean theorem" is the subject and "is true" is the predicate, while in the second "that the climate is getting warmer" is the subject and "is true" is the predicate, and in the third the word "it" is the main clause's subject (in that "it" plays the role of a so-called "dummy subject") and "is true" is its predicate. From the point of view of several interpretations that show up in contemporary linguistic theories, it is also the case that all of these can be treated as subject-predicate sentences. Here, it should be noted that in the explanations that follow, the terms "subject" and "predicate" denote grammatical functions, whereas the terms "nominal phrase" and "verbal phrase" capture grammatical categories (Chomsky 2015, 73). In the first sentence, the expression "the Pythagorean theorem," which is a nominal phrase, is the subject, and the expression "is true," which is a verbal phrase, is the predicate. In the second one the expression "that the climate is getting warmer," which is a complementizer phrase, is the subject, and the expression "is true," a verbal phrase, is the predicate. In the third sentence the word "it," which is a nominal phrase, is the subject of the main clause, and "is true," a verbal phrase, is its predicate; the word "it" plays the role of a cataphor in this sentence, with the whole grammatical construction being an instance of what is known as "*it*-extraposition" (see Huddleston 1984, 451). (It is worth noting that some philosophers of language claim that sentences such as "It is true that Wittgenstein appreciated Frege very much" are in some sense derivative of sentences such as "That Wittgenstein appreciated Frege very much is true" [see Dummett 2000; Horwich 1990; Künne 2003; Parsons 1993].) I wish to strongly emphasize here that the following considerations pertaining to truth-expressions and the sentences in which they occur are not aimed at questioning the appropriateness, in some contexts and for certain purposes, of treating truth-expressions as predicates and the sentences in which they occur as subject-predicate sentences. That is, if it is useful for the purposes of a syntactic theory to treat

the expression "is true" as a predicate, and the sentences in which it occurs as subject-predicate sentences, then this expression, and sentences of this kind, should be treated in this way. However, the above observation need not entail the conclusion that these expressions should be treated in this way in every possible description of language use.

It seems that in at least some sentences truth-expressions play the role of operators, not predicates. One of the examples of such sentences is "It is true that Wittgenstein appreciated Frege very much." Here I will not discuss the reasons why viewing the role of truth-expressions in this type of sentences as operators is more adequate than viewing them as predicates—I'll try to do so later. Nevertheless, I would like to draw attention here to a certain analogy between truth-expressions and modal expressions, which may help us understand why truth-expressions can be treated as playing the role of an operator. From the point of view of logic, it is usually assumed that in sentences such as "It is necessary that 2 + 2 = 4," "It is possible that people will live on the Moon" the modal expressions "it is necessary that," "it is possible that" play the role of an operator, not a predicate. This means that these sentences are analysed as follows:

It is necessary that / 2 + 2 = 4
It is possible that / people will live on the Moon.

At first glance, the expression "it is true that" plays a role in the example given above that is analogous to the role played by the expressions "it is necessary," "it is possible" in such modal sentences as those presented here. So if the above-mentioned analogy is accurate, then from the point of view of logic, at least in some cases, truth-expressions play the role of an operator. The question of when truth-expressions function as a predicate and when they act as an operator will, of course, be discussed in more detail in subsequent sections of the text.

3. Wittgenstein's Conception of Grammatical Investigations

These loose remarks concerning how we should go about describing the role or use of truth-expressions require further elaboration. Firstly, we need to point out what the purpose of the proposed description is. Secondly, it should be explained why, for some purposes, interpreting truth-expressions as predicates may not only fail to elucidate the functioning of our language, but actually also obscure it and in this way become a source of philosophical problems.

As I have noted above, the approach to the question of the functioning of truth-expressions adopted here is inspired by Wittgenstein's approach

to philosophical problems. The latter points out in *Philosophical Investigations* that philosophy aims at such a description of language use as will serve to (dis)solve philosophical problems (see Wittgenstein 2009, § 109). It should be added that according to Wittgenstein, solving philosophical problems does not consist in formulating philosophical theses or theories, but in making the problems themselves vanish (see 2009, § 133).

Although the entirety of my considerations are, in principle, focused on the second task, I would like at the outset to make some more general comments on Wittgenstein's method for dealing with philosophical problems. According to Wittgenstein, the main source of philosophical problems is our urge to misunderstand the "workings of our language" (see 2009, § 109, § 111). This inclination is manifested, amongst other things, by the fact that "our ordinary forms of language easily make us overlook" (Wittgenstein 2009, § 132) important differences in respect of the use of expressions of our language. Firstly, this statement can be understood as a reminder that "the uniform appearance of words when we hear them in speech, or see them written or in print" (Wittgenstein 2009, § 11) does not permit us to draw the conclusion that their use is also uniform, or even just similar. Secondly, it can also be interpreted as reminding us that sameness of grammatical structure (in the sense of this term employed in linguistics), regardless of whether it be surface or deep structure, can conceal fundamental differences with regard to use—that is, grammatical differences in Wittgenstein's sense of the term. It is worth adding here that Wittgenstein had already pointed out in the *Tractatus* that sameness at the level of our ordinary linguistic forms can obscure fundamental differences in respect of logical form: that is, differences in the logico-syntactic application of signs (see Wittgenstein 1922, 3.327; Hacker 2021, 22). Moreover, he seems in that earlier work to have assumed that the best way to gain insight into the logical form of ordinary language propositions[1] is to translate them into a proper logical notation (see Hacker 2021, 22). Later, he abandoned the second conviction, because he realized that the logical (i.e. grammatical, in Wittgenstein's sense of the term) form of a proposition is not fully determined by structural properties of the sentence used to express that proposition, even if the latter is formulated in proper logical notation: according to the later Wittgenstein, what establishes the logical (grammatical) form of an expression is its use,[2]

1. Here, I am employing the term "proposition" as it is used in English translations of the *Tractatus*: a proposition is, roughly speaking, a meaningful sentence (see Wittgenstein 1922, 3.12).

2. "Grammar describes the use of words in the language" (Wittgenstein 1974, 60).

not merely its structural features.[34] Of course, this does not mean that in his later writings he was claiming that appealing to various logical notations in order to (dis)solve philosophical problems is unjustified or useless: rather, he was simply acknowledging that translating the expressions of our language into logical notation will not give us the sort of insights into the functioning of our language that would automatically enable us to (dis) solve philosophical problems.

In what sense, for Wittgenstein, can sameness of grammatical structure obscure fundamental differences in regard to use? Answering this question will allow us to understand, at least to some degree, the Wittgensteinian conception of grammatical investigations. In my opinion, the best way to approach this is to look at a few examples. Let us consider the following pairs of sentences: "Zero is a number" and "Biden is a president"; "Redness is a colour" and "Justice is an illusion"; "Mark killed a dragon" and "Mark looked for a dragon"; "This rose is red" and "This proposition is true." Sentences belonging to each of those pairs have the same grammatical structure, but their uses are fundamentally different. The first sentence of the first pair is used to express an analytic proposition (of course, provided that one does not completely reject the analytic/synthetic distinction), whereas the second undoubtedly expresses a synthetic one. Moreover, even if one acknowledges that both words that are subjects in the sentences of the first pair serve to stand for something, they do so (i.e. they stand for what they stand for) in completely different ways. It seems that in the case of the word

3. "The names I give to bodies, shapes, colours, lengths have different grammars in each case." (Wittgenstein 1974, 63).

4. I try to develop this interpretation of the later Wittgenstein's approach to the relationship between the logical (grammatical) form of a proposition and the structural features of the sentence used to express that proposition in the next paragraph. To clarify the matter, I refer to a few examples. An anonymous reviewer pointed out that the interpretation I presented is controversial, because in his opinion: "It may be on the contrary argued that the later Wittgenstein believed that logical/grammatical form of a proposition is determined by structural properties of the sentence and that's one of the reasons why he abandoned efforts to represent sentences in abstract logical notation." This suggestion is undoubtedly interesting, but I find it difficult to fully agree with it. Wittgenstein, for example, in § 21 and § 49 of the *Investigations*, points out that the same sequences of words, and therefore the linguistic expressions with the same structure, can be used in fundamentally different ways, and thus their grammar (in Wittgenstein's sense of the word) can be different. The words "Five slabs" can sometimes function as an order and sometimes as a statement (2009, § 21); the same signs can sometimes be sentences and sometimes words (see 2009, § 49). The anonymous reviewer is, of course, right that Wittgenstein, in his grammatical investigations, also takes into account the structural features of expressions, but I think the content of § 664 of the *Investigations* shows that he does not assign them a key, or at least exclusive, role in determining the logical (grammatical) form (see 2009).

"Biden" the term "standing for" applies most straightforwardly, because this sign can be attached to the designated person as a label. (Such things happen, for example, at conferences or official meetings [see: Wittgenstein 2009, § 15].) Of course, we can also say of the word "zero" that it stands for the number zero, but in this case there is no sense in even trying to attach this label to some object designated by the word. It should be emphasized that we do not learn to use the word "zero" by pointing to a specific object, but by mastering arithmetic. A grammatical difference—in Wittgenstein's sense of the term—between the sentences belonging to this pair is also manifested by the fact that if one substitutes the words that play the role of subjects in these sentences for each other, one will obtain nonsensical sentences. The sentences "Biden is a number" and "Zero is a president" will not have any sense unless a new meaning is ascribed to their constituents.[5] In the case of the second pair, the differences with respect to the use of these sentences are as follows: the first sentence seems to be an analytic one, while the second is not, and, moreover, if one replaces the word "redness" in the first one with the word "justice," one obtains nonsense. The sentences belonging to the third pair also differ from each other in respect of their grammar in Wittgenstein's sense of the term: the first can only be true if a certain dragon exists, while the truth of the second does not require this.

In my view, this very short discussion of the above examples suffices to show that expressions which have the same grammatical structure in the standard linguistic sense of the term can be used in fundamentally different ways. Simplifying greatly, one can say that sameness of grammatical structure is something that can conceal certain categorial differences.[6] For example, although the words "zero" and "Biden" are singular terms, they cannot be said to belong to the same category in the way that, say, the words "Biden," "Frege" and "Coltrane," or the words "zero," "one" and "million," can. Their belonging to different categories manifests itself in, amongst other things, the fact that in a given context the result of replacing the one term with the other is nonsense. It also shows up in the form of radical differences in the processes involved in learning them, and in

5. These sentences are nonsensical in my opinion. However, the question of the status of such sentences is controversial—some philosophers treat them not as nonsense, but as false (Prior 1976; Magidor 2009). I will not argue here that they are nonsense, but I would like to point out that even if we treat them as false, their use is fundamentally different from the use of the sentences "Biden is a president" and "Zero is a number" because sentences like "Biden is a number" are treated as either necessarily false or manifestly and indisputably false, and the former are not regarded as such.

6. I am using the term "categorial" here in the sense of Ryle (1938).

the practices of using them. For example, in the case of names of persons or observable objects one of the criteria for one's being able to use them is one's capacity to pick out a designatum of some given name when it is accessible to one's senses, whereas in case of numerals it is obvious that this criterion does not apply.

Of course, one can point to a group of objects of a given cardinality, but even if one were to call this pointing to a number, it would be pointing to the designatum of a given name in a completely different sense than in the case of pointing to that of a name standing for a person: pointing to the designatum of the singular term "Biden" consists in pointing to one and the same person, Biden, whereas pointing to, say, three apples or three chairs, is not pointing to the designatum of the singular term "3," but at most pointing to something that can help us understand how to use this singular term.

It should be emphasized that, according to Wittgenstein, grammatical investigations are conceptual investigations and therefore differ fundamentally from factual investigations (2009, § 90; 1970, § 458). In the case of conceptual investigations, problems are solved not by discovering new facts, "but by assembling what we have long been familiar with" i.e. by "an insight into the workings of our language" (Wittgenstein 2009, § 109).[7]

To sum up, the goal of grammatical investigations, in the Wittgensteinian sense of the term, is not to formulate a theory that provides a collection of rules that generate a set of all and only those expressions of a given language that are well-formed. The purpose of these investigations is, instead, to provide the kind of elucidations of the use of various expressions that will make it possible to dissolve various philosophical problems and puzzles—where these elucidations, moreover, need not only concern the structure of expressions that the philosopher deals with:

> The important difference is in the aims for which the study of grammar are pursued by the linguist and the philosopher.… Our object is to get rid of certain puzzles. The grammarian has no interest in these; his aims and the philosopher's are different. (Wittgenstein 2001, 31)

7. Hacker draws attention to this aspect of Wittgenstein's philosophy: "Conceptual problems are *toto mundo* distinct from factual, scientific, ones, and cannot be resolved by scientific advances, but only by clarification of the use of words" (Hacker 2021, 196).

4. Frege, Wittgenstein, Prior, and Brandom on Truth

As I have already noted, one of my principal objectives here is to show that while truth-expressions are very often treated from the point of view of studies of grammar in the linguistic sense of the term as playing the role of a predicate, investigations that can be considered grammatical in Wittgenstein's sense lead to the conclusion that in many contexts these expressions should be treated as operators rather than predicates. Before I present my own approach to truth-expressions, according to which in many contexts they play the role of an operator, and where this has the positive consequence that certain philosophical problems are then dissolved, I would like to recall the views of four philosophers who were either sceptical about treating truth-expressions as predicates referring to a certain property, or claimed that, in principle, they play the role of an operator. I have in mind Frege, Wittgenstein, Prior and Brandom. I am aware that this selection is to a certain degree arbitrary: for example, one could also invoke, in this regard, the views of Ramsey, Grover and C.J.F. Williams (Ramsey 1927; Grover et al. 1975; Williams 2009).

A detailed analysis of Frege's approach to truth would require the writing of a separate article, at least, because according to him the concept of truth plays a foundational role in logic (Frege 1984b, 351); I will therefore just offer some comments on this topic here. I would like to focus exclusively on his remarks that suggest that treating being true as an ordinary property, and truth-expressions as ordinary predicates, is dubious to say the least.[8] Frege, in *Thoughts*, points out that from the linguistic point of view the word "true" is a word for a property (Frege 1984b, 352), but he also adds that being true is something quite special, and that one can legitimately doubt whether it is an ordinary property (Frege 1984b, 354–5). So why regard this as questionable? Being true would be an ordinary property if the "relationship" between a thought and being true were like that between a subject and a predicate. However, this is not the case, because when one utters words that are a concatenation of a sentence expressing a given thought with the expression "is true" one does not assert anything more than if one just uses the sentence itself (Frege 1984a, 164). In a letter to Russell, he explicitly claims that he does not view the word "true" as an ordinary predicate (see Frege 1980, 163).

8. It is worth adding that, for example, Greimann suggests that Frege expresses the concept of truth by means of the judgement-stroke, which should be interpreted as a truth-operator, not a predicate (Greimann 2000).

Wittgenstein approaches this in a similar way. He claims in the *Tractatus* that treating truth and falsehood as if they were ordinary properties is tantamount to a misunderstanding:

> One could e.g. believe that the words "true" and "false" signify two properties among other properties, and then it would appear as a remarkable fact that every proposition possesses one of these properties. (1922, 6.111)

The meaning of the above quotation can be clarified by showing that, according to Wittgenstein, the concept of a proposition cannot be grasped without grasping the concepts of truth and falsehood, and this is manifested in the fact that the values of propositional variables can only be truth-values, i.e. truth and falsehood (one can add that a reverse relationship also holds, i.e. the concepts of truth and falsehood cannot be grasped without grasping the concept of a proposition, and this is manifested, among other things, in the fact that in the schemas "it is true that *p*," "it is false that *p*" only sentences can be substituted for the variable "*p*"). Truth and falsehood are not certain contingent properties of propositions: rather, they are constitutive of what a proposition is (Diamond 2003). Moreover, Wittgenstein seems to claim that the expressions "is true" and "is false" do not incorporate real verbs into propositions (see 1922, 4.063).

His approach to the problem of truth does not fundamentally change in *Investigations*.[9] In §§ 136–7 we encounter the suggestion that truth and falsehood should not be treated as features that fit what a proposition is. The concepts of truth and falsehood rather belong to the concept of a proposition (this idea is extensively discussed by Bartunek [2019]). That is, the fact that sentences of a certain type, when concatenated with the expressions "is true" or "is false," form meaningful wholes means that sentences of this type, when employed assertorically, express propositions (see Wittgenstein 2009, §§ 136–137). It is worth adding that according to Wittgenstein, what a proposition is is also determined by the fact that the expression "this is how things are:" can be put in front of it (see 2009, § 137). These remarks show that in his opinion the expression "is true" plays an analogous role to the expression "this is how things are:"—where the latter, of course, is not used as a predicate, but rather serves to indicate what is asserted. This expression can also be recognized as an operator that, when applied to a proposition, gives as a result the very same proposition. I shall discuss these suggestions in more detail in due course.

9. This issue is discussed by Baker and Hacker (2005, 349–55).

Another philosopher who concerned himself with the question of how sentences of the form "It is true/false that *p*" should be interpreted was Arthur Prior. It is worth adding that Prior's analysis of the logical form of this kind of sentence is connected with his analysis of the logical form of sentences of the form "X thinks/says/believes that *p*." The former, according to his analysis, are composed of an operator (a connective) and a sentence, while the latter are composed of names, "predicates at one end and connectives at the other," and sentences (see Prior 1971, 19). That is, for Prior, sentences such as "It is true that Wittgenstein appreciated Frege very much" and "Copernicus believed that the Earth revolves around the Sun" should rather be decomposed into "It is true that / Wittgenstein appreciated Frege very much" and "Copernicus / believed that / the Earth revolves around the Sun" than into "It / is true / that Wittgenstein appreciated Frege very much" and "Copernicus / believed / that the Earth revolves around the Sun." Prior claims that uttering the first sentence does not serve to predicate the property of truth of a certain object—namely, a proposition—that is signified by the expression "that Wittgenstein appreciated Frege very much," and uttering the second sentence does not serve to assert the obtaining of the relation of believing between Copernicus and a certain proposition designated by the expression "that the Earth revolves around the Sun."

Prior's analysis of these kinds of sentences leads to the conclusion that propositions are logical constructions, and that statements which seem to be about propositions are, in fact, about something else (see 1971, 19). So, he is questioning the belief that truth-expressions are in principle predicates, and that truth and falsehood are properties of propositions. However, it is worth adding that he does not deny that truth-expressions are predicates in certain contexts and that they refer to genuine properties in these contexts. The expressions "is true" and "is false," when predicated of sentences, refer to genuine properties: "The truth and falsehood with which Tarski is concerned are genuine properties of genuine objects, namely sentences" (Prior 1971, 98).

Yet another philosopher who questions treating truth-expressions as predicates and truth as a property is Robert Brandom. In his writings, Brandom develops the so-called "prosentential theory of truth" originally formulated by Grover, Belnap and Camp (1975). According to the original version of the theory, truth-expressions should be treated as parts of prosentences, where these parts are not semantically independent, and one of the examples of prosentences is the expression "that is true." Prosentences serve several purposes in a language: they are used to confirm what was said earlier, or to formulate generalizations. Brandom's original contribution

to the development of the prosentential theory of truth consists in his showing that the deflationary intuitions that motivate acceptance of the theory are not to be expressed in terms that involve claiming that "is true" is just a predicate of a specific kind: namely, one that refers to no property. The expression is, rather, only apparently a predicate, and is, in fact, a prosentence-forming operator.[10] When one treats it this way, the question of whether it refers to a certain property and, if so, what the nature of this property is, actually becomes meaningless:

> Notice, however, that this argument depends on treating "… is true" as a predicate. If it is, then since that expression is used to make claims and state facts, it must, on deflationary accounts, be taken to express a property. But the essence of the anaphoric approach to truth talk is precisely to take issue with this grammatical presupposition. According to those accounts, "… is true" expresses a prosentence-forming operator. (Brandom 1997, 147)

5. How Should Sentences Containing Truth-Expressions be Classified?

Having outlined the difference between Wittgenstein's conception of grammatical investigations and the linguistic approach to the study of grammar, and after recalling the most important—from the perspective of the issues considered in this text—remarks on truth made by Frege, Wittgenstein, Prior, and Brandom, I shall now move on to a description of the grammar, in Wittgenstein's sense of the term, of truth-expressions. Of course, this description is based on certain thoughts or threads contained in the works of the above-mentioned authors. I will start by giving a few examples of sentences in which these expressions are used:

1. The Pythagorean theorem is true.
2. It is true that Kant read Hume.
3. That the Earth revolves around the Sun is true.
4. Ann asserts truly that Glenn Gould recorded the Goldberg Variations in 1981.
5. Some of Peter's beliefs are not true.
6. The sentence 'London is not always foggy' is true.
7. The first sentence of Frege's paper *On Concept and Object* is true.

These sentences can be classified in various ways. Firstly, they can be divided into ones that are or are not metalinguistic: 6 and 7 have such a character, while the remainder do not. Secondly, they can be categorized

10. Brandom's approach to truth is defended and developed by Salis (2019b).

as general or singular, with only 5 being general, and the rest singular. Thirdly, they can be divided into those where what is true is displayed in the very sentences themselves, and those where it is not displayed there: 2, 3, 4 and 6 are of the first sort, while 1, 5 and 7 are of the second kind. And, finally, they can be classified as sentences that are standardly interpreted as involving the predicate "is true," and those whose surface form can be taken to mean that they do not involve the predicate "is true," but rather such truth-expressions as do not serve to predicate certain properties of certain objects: here, sentences 1, 3, 5, 6 and 7 will belong to the former group, 2 and 4 to the latter one (see Mulligan 2010). In this text, I focus mainly on issues connected with this fourth classification; however, these cannot be even summarily addressed without also raising certain questions that pertain to the other classifications.

The fourth of the above divisions proceeds on the basis of whether a given truth-expression in a given context is used to predicate a certain feature of a certain object, or in some other way. According to this classification, some truth-expressions play the role of a predicate, but there are other ones which do not play this role. As has been pointed out by certain authors (Mulligan 2010; Prior 1971), the truth-expressions that occur in sentences like 2 and 4 should be recognized as operators. In 2, the truth-expression is "it is true that," while in 4 it is the adverb "truly." We can assume that 2 is formed by joining the operator "it is true that" with the sentence "Kant read Hume." The matter is more complicated, though, in the case of 4. "Ann asserts truly that Glenn Gould recorded the Goldberg Variations in 1981" can be analysed as a conjunction of the sentences "Ann asserts that Glenn Gould recorded the Goldberg Variations in 1981" and "It is true that Glenn Gould recorded the Goldberg Variations in 1981." The latter is formed by linking the operator "it is true that" with the sentence "Glenn Gould recorded the Goldberg Variations in 1981."

6. Elucidating the Use of the Truth-Expressions Occurring in Sentences 1–7

Here, I would like to acknowledge that various doubts can be raised with regard to my fourth classification of truth-expressions. First of all, one might claim that all truth-expressions are, in fact, predicates, and that therefore none of them are operators. Secondly, one could insist that things are the other way round: i.e. that all truth-expressions are operators, and none of them are predicates. Thirdly, one may doubt whether truth-expressions should be classified as operators and predicates in exactly the way suggested in the foregoing paragraphs.

Certain authors hold that every sentence in which a truth-expression occurs either consists of a predicate and a term designating a proposition (or a sentence, if the whole is a metalinguistic sentence), or consists of a quantifier, a predicate, and a propositional variable (or an individual variable, if the whole is a metalinguistic sentence). On this view, sentence 2, "It is true that Kant read Hume," is a stylistic variation of "That Kant read Hume is true" (see Künne 2003, 351; Horwich 1990, 16). It should be added that the form of the latter sentence is treated as more basic, in that it is taken to reveal the logical form of the former sentence in a more perspicuous way. Several arguments in support of this analysis have been presented. First of all, it has been pointed out that contemporary linguistics treats such expressions as "that Kant read Hume" and "is true" as grammatical units (CP—a complementizer phrase, and VP—a verbal phrase), whereas according to contemporary linguistics the expression "it is true that" is not treated as a grammatical unit. These expressions are treated in this way because, *inter alia*, the result of adding the phrase "—as is well known—" to the sentence "It is true that Kant read Hume" is the sentence "It is true—as is well known—that Kant read Hume," not the sentence *"It is true that—as is well known—Kant read Hume" (see Künne 2003, 69). Künne, meanwhile, presents another argument in favour of the analysis now under consideration:

> Consider
> (A) It is true that his paper is clever, but her objection is *also* true.
> We can make literal sense of the "also" if it is preceded by another application of the predicate "is true" in the first half of the sentence. But on the operator reading we can find no predication of "is true" there. (Künne 2003, 351)

Let us examine these arguments. As regards the first of them, it is worth noting that if the analysis of sentences of the type "It is true that Kant read Hume" as being composed of the operator "it is true that" and the sentence "Kant read Hume" is not a linguistic analysis of the grammatical structure of this sentence, then the fact that this analysis is not compatible with the currently dominant theoretical approaches to grammar in contemporary linguistics does not settle the dispute. It should be added that the fact that one cannot insert a parenthetical expression such as "—as is well known—" after "that" is also not a decisive argument in favour of the view that truth-expressions always function as predicates, because such sentences as "It is true—as is well known—that Kant read Hume" can be analysed as being composed of two operators and a sentence. That is, this sentence can be analysed in the following way:

As is well known / it is true that / Kant read Hume.

One can reply to the second argument as Mulligan did:

> According to the alternative account of "also" in (A), anyone who understands the first half of (A), that is,
>
> It is true that his paper is clever,
>
> knows that
>
> It is true that his paper is clever iff <his paper is clever> is true
>
> and that
>
> If (<his paper is clever> is true), then (<his paper is clever> is true because it is true that his paper is clever).
>
> The predicate "is true" is no part of
>
> It is true that his paper is clever,
>
> but *is* readily available to anyone who understands the sentence (Mulligan 2010, 578–9).

This argument, however, can also be questioned in a different way: for example, by demonstrating that the second truth-expression in (A) (i.e. "It is true that his paper is clever, but her objection is *also* true") also plays the role of an operator. Such an interpretation of (A) can have the following form:

> It is true that his paper is clever, but for some p, such that her objection says that p and, for every q, if her objection says that q, then the proposition that q will be the same (i.e. the same proposition) as the proposition that p ($p \equiv_i q$), and it will also be true that p.

In this paraphrase, the variables "p" and "q" are propositional ones: that is, they should be substituted with sentences, and their values are propositions. The quantification here should be treated as neither objectual (in that the values of variables of this type will not be objects of any kind) nor substitutional (Hugly and Sayward 1996, 209; Prior 1971, 35). (Later, I shall return to the issue of how we should interpret quantification of this kind in such contexts, though I do not intend to discuss it at any great length.) "The proposition that … is the same proposition as the proposition that …" is an intensional sentence-forming functor which has as its arguments sentences. (The sign "$\equiv_i$" is used here to symbolize this connective.) The role of this expression in our language and, especially, in the analysis of statements containing truth-expressions, is discussed by Williams (Williams 2009, 29–41; see Prior 1971, 53–6). Simplifying greatly, the meaning

of this connective can be explained as follows: a sentence that is the result of joining two sentences by this connective will be true if and only if in any context C, substituting one of these sentence with the other does not entail a change in the truth-value of the context C. (In short, both sentences should be interchangeable *salva veritate* in every context.) However, certain contexts where these sentences occur in quotation marks represent exceptions to this. For example, it is generally speaking true that the proposition that the Second World War started in 1939 is the same proposition as the proposition that World War II began in nineteen thirty-nine, but in such a specific context as "The sentence '…' consists of x letters," the substitution of the sentence "The Second World War started in 1939" with the sentence "World War II began in nineteen thirty-nine" will change the truth-value of the whole context (see Prior 1971, 56–61). Still, I would like to emphasize again that the above explanation of the meaning of this connective assumes a highly simplified view as regards the identity of propositions. One can have reasonable doubts about whether it is possible to give such a definition as would enable one to decide the following question in every context: do any two given sentences express the same proposition or two different propositions? My approach to this is similar to that of Travis (2000, 141–4). According to him, one can give an unambiguous answer to a question concerning the identity of two propositions only if the context in which the question has been raised is determined. For example, the two sentences "Mark drank two big (half-litre) beers in the pub" and "Mark drank one litre of beer in the pub" express the same proposition, in the context of considering the amount of alcohol drunk by Mark, but might express two different propositions in the context of a question about what Mark ordered in the pub. (At the end of this somewhat sketchy discussion of the intensional connective in question, I should perhaps add that its use is justified, no matter whether Travis' approach to propositions or some other is the correct one.)

Now I shall present another possible criticism of the division of truth-expressions into predicates and operators. This reflects the conviction that all truth-expressions are in principle operators, not predicates. On this view, the truth-expressions that occur in sentences 6 ("The sentence 'There is not always a fog in London' is true") and 7 ("The first sentence of Frege's paper *On Concept and Object* is true") are also operators. I shall not try to answer this objection to the above classification of truth-expressions at this juncture; nevertheless, some things that will be said later on can be treated as furnishing such an answer.

One can make yet another objection to the division of truth-expressions into predicates and operators presented above: namely, it can be argued that the line of division should be drawn in a different way, with not only the truth-expressions occurring in 2 and 4 being classified as operators, but also those in 1, 3 and 5. Of course, this means that a genuine predicate only occurs in sentences 6 and 7. Such an elucidation of the grammar—in Wittgenstein's sense of the term—of the truth-expressions in 1–7 seems adequate to me; however, one should note that adopting this point of view on the grammar of such truth-expressions will prompt one to ask certain questions. How should one explicate the grammar of 1, 3 and 5? How should propositional quantification be understood? What is the function of the word "that" in sentences 1–5 and their paraphrases? Why should one treat the truth-expressions occurring in 6 and 7 as predicates rather than operators? How should this predicate be understood? Of what kind of objects can the property denoted by this predicate be predicated?

According to this proposal, the grammar of the sentence "The Pythagorean theorem is true" can be explicated as follows:

> For some p, such that the Pythagorean theorem asserts that p, and for every q, if the Pythagorean theorem asserts that q, then q will be the same proposition as p ($q \equiv_i p$), and it will be true that p.[11]

In the sentence above, "p" and "q" are, of course, propositional variables. Moreover, according to the explication proposed here, expressions such as "the Pythagorean theorem asserts that p" should not be interpreted like expressions of the type "Peter asserts that p." That is, treating the latter expression as being composed of the word "Peter," playing the role of a genuine name, the expression "asserts that," which is a predicate at one end and a connective at the other (see Prior 1971, 19; Recanati 2000, 30–3), and the variable "p," seems appropriate, whereas—in my opinion—the former expression should not be treated in an analogous fashion. It seems that from the point of view of grammar—in Wittgenstein's sense of the term—the first of these two can be interpreted as composed of the phrase "the Pythagorean theorem asserts that" and the variable "p," rather than the name "the Pythagorean theorem," the phrase "asserts that," and the variable "p." In my view, the expression "the Pythagorean theorem" does not play the role of a genuine name (it is not a complete expression in Russell's sense of

11. A similar account of the sentence "What Percy says is true" is, for example, presented by Williams (2009, 38).

the term), but is, in fact, a part of the operator "the Pythagorean theorem asserts that." This expression is an intensional operator: the result of its application to sentences expressing the Pythagorean theorem is truth, and the result of its application to other sentences is falsehood. One can argue for this analysis in the following terms. Let us consider the two sentences "The Pythagorean theorem asserts that the square of the hypotenuse is equal to the sum of the squares of the other sides" and "Peter asserts that the square of the hypotenuse is equal to the sum of the squares of the other sides." In the latter, the phrase "asserts that the square of the hypotenuse is equal to the sum of the squares of the other sides" is a genuine predicate that is used to ascribe a certain property to Peter. Moreover, if one substitutes this predicate with other predicates in this sentence, one will obtain other meaningful sentences. By contrast, in the former, the expression "asserts that the square of the hypotenuse is equal to the sum of the squares of the other sides" does not serve to ascribe a certain property to the object designated by the name "the Pythagorean theorem." Instead, the sentence is being employed to state the content of the Pythagorean theorem. Thus, its grammar can be made explicit in the following way:

> The Pythagorean theorem: the square of the hypotenuse is equal to the sum of the squares of the other sides. (The word "asserts" has been omitted as it is completely redundant.)

This explication enables one to see that in the contexts considered above the grammar (in Wittgenstein's sense of the term) of the expression "the Pythagorean theorem" is different from the grammar of the word "Peter." If one assumed that it is the same, one would then obtain some sort of nonsense like the following:

> Peter: the square of the hypotenuse is equal to the sum of the squares of the other sides.

This difference between the role of the word "Peter" and that of the expression "the Pythagorean theorem" is also shown by the following explication of the grammatical form (in Wittgenstein's sense of the term) of the sentence "Peter asserts that the square of the hypotenuse is equal to the sum of the squares of the other sides":

> Peter asserts: the square of the hypotenuse is equal to the sum of the squares of the other sides.

In this sentence, the expression "asserts: the square of the hypotenuse is equal to the sum of the squares of the other sides" plays the role of a predicate and therefore the word "Peter" is used as a name, whereas in the expression "The Pythagorean theorem: the square of the hypotenuse is equal to the sum of the squares of the other sides" the expression ": the square of the hypotenuse is equal to the sum of the squares of the other sides" does not seem to play the role of a predicate and therefore from the point of view of Wittgensteinian grammar the expression "The Pythagorean theorem" does not seem to be used as a name. It is more natural to make the following division of the expression considered above:

> The Pythagorean theorem: \ the square of the hypotenuse is equal to the sum of the squares of the other sides.

According to this division, the expression "The Pythagorean theorem:" seems to act as an operator applied to the sentence "The square of the hypotenuse is equal to the sum of the squares of the other sides."

These explications also help one to see that the grammar of the expression "asserts that the square of the hypotenuse is equal to the sum of the squares of the other sides" in the sentence stating the content of the Pythagorean theorem differs from its grammar in the sentence about Peter. This expression can be substituted with various predicates in the latter, whereas, if one assumes that the above explication is correct, it will not be possible to make such substitutions in the former.

The grammar of the sentence "That the Earth revolves around the Sun is true" should be represented, of course, in this way:

> It is true that the Earth revolves around the Sun.

Meanwhile, in the case of an explication of the sentence "Some of John's beliefs are not true," we have to invoke propositional quantification. It seems that such an explication can take the following form:

> For some *p*, John believes that *p*, and it is not the case that it is true that *p*.

In this section I discussed various possible ways of classifying truth-expressions occurring in the types of sentences given above. According to one classification, truth-expressions always play the role of a predicate. According to another, these expressions always play the role of an operator. In my view, both of these extreme approaches are wrong—I tried to

show that truth-expressions occurring in sentences 1–5 play the role of an operator, and truth-expressions occurring in sentences 6–7 play the role of a predicate.

7. Is Propositional Quantification Problematic?

One of the objections that can be made to the above elucidation of the role played by truth-expressions in such sentences as 1 ("The Pythagorean theorem is true") and 5 ("Some of Peter's beliefs are not true") is that either genuinely propositional quantification (which is an instance of higher-order quantification) only appears to help explain the meaning of general sentences about thoughts, beliefs and statements (see Horwich 1990, 4; Soames 1999, 41–6), or it is only seemingly intelligible (see Quine 1963).

Before I turn to a discussion of this objection, and a possible answer to it that seems right to me, it ought to be noted that adherents of similar approaches to the grammar (in Wittgenstein's sense) of truth-expressions have widely discussed the subject of propositional quantification in the context of interpreting sentences in which these expressions occur (Prior 1971; Williams 2009; Grover 1972; Hugly and Sayward 1996). Propositional quantification is a form of quantification in which quantifiers bind propositional variables, and propositional variables are, in turn, such variables as can be substituted with sentences.

What are the reasons for questioning the possibility of an explanation of the meaning of general sentences about thoughts, beliefs and statements by means of propositional quantification? The basic reason is that the meaning of quantification of this kind can be explained only by specifying what expressions can be substituted for propositional variables. (Such a view seems to be adopted by Horwich [1990, 4, 32].).[12] That means—according to critics—that propositional quantification can only be understood as substitutional quantification. This, however, in their opinion shows that an appeal to propositional quantification does not allow one to explain the meaning of general sentences about thoughts, beliefs, statements, etc., because these general sentences may concern such thoughts, beliefs and statements as cannot be expressed by the sentences of one's language—that is, the language to which these general sentences belong. (Such a line of criticism of substitutional quantification is considered by Hugly and Sayward [1996, 243–6].) As an example of such a general sentence, one can

12. Quine, it should be noted, suggests that substitutional quantification, in contrast to objectual quantification, allows other kinds of expressions as substituents, apart from names (Quine 1969, 106).

give "All statements which logically follow from true statements are also true." Critics of the substitutional interpretation of propositional quantification would argue that one cannot reduce the sense of this sentence to the statement that all substitutions of the appropriate scheme ("If it is true that *p*, then, if *q* logically follows from *p*, it is true that *q*") are true, because there are many propositions which cannot be expressed in any actual language. Moreover, according to Hugly and Sayward (1996, 296–302), a substitutional interpretation of propositional quantification misrepresents the truth-conditions of quantified sentences, because it assumes that one of these conditions is the existence, or possibility of existence, of certain linguistic expressions. Such an interpretation assumes, for example, that one of the truth-conditions of the sentence "Some atoms of gold are located in Austria" is that these atoms can be named, and yet it is clear that this sentence does not involve that issue at all.

As I have already indicated, propositional quantification is also criticized as being incomprehensible. This point can be supported by several arguments. Firstly, one can argue that propositional quantification must be substitutional quantification, and the latter is itself impossible to comprehend. Arguments for the incomprehensibility of substitutional quantification have been formulated by van Inwagen (1981), but I will not present them here, as this would require a lengthy digression. Secondly, one can question propositional quantification on the basis of the belief that there are no expressions in natural languages that play an analogous role to that performed by the propositional variables in formal calculi (van Inwagen 2002, 222), and that the formal constructions which serve to explain the functioning of natural languages are only intelligible in so far as there exist some counterparts of these constructions in natural languages. (This line of argument is discussed by Hugly and Sayward [1996, 246–58].)

In my view, as a first point when seeking to respond to this criticism of propositional quantification one should note that certain constructions fulfilling the role of propositional quantification occur in many of the ordinary sentences we use. This fact demonstrates conclusively that propositional quantification is intelligible. Thus, the above arguments can at most show that we do not have any satisfactory explanation of propositional quantification. (This question is discussed by Båve [2013], who rightly points out that there is no greater problem with understanding propositional quantification than with understanding first-order quantification.) However, this conclusion may also raise various doubts. Firstly, it is not at all obvious that propositional quantification requires any explanation (see Prior 1971, 35). Secondly, it is not clear what criteria such an explanation must meet in order

to be deemed satisfactory. In my reply to the objections presented above, I shall limit myself to two points: I will try to show the special character of propositional quantification, and I will attempt to demonstrate that there are expressions of this sort in natural languages that play a similar role to propositional variables.

The first objection, so it seems, assumes that propositional quantification can be understood either as referential or as substitutional. If one were to suppose that propositional quantification was to be interpreted as objectual, then sentences would have to be recognized as specific names that designate objects of a peculiar kind. This, in turn, would mean that there would be no need to distinguish propositional quantification as a specific kind of quantification. So, is it possible to understand propositional quantification as referential, but not objectual? Of course, this would only be possible if sentences referred not to objects, but to some other kind of entity. However, what could these entities be? An answer that, for certain reasons, seems quite natural—namely, that according to which these entities could be truth-values—is completely unsatisfactory for other reasons. Firstly, it is not satisfactory because, for example, Frege treated truth-values as objects. Secondly, an explanation of propositional quantification solely in terms of truth-values ascribable to propositional variables does not allow one to elucidate the meaning of the sentences in which such quantification occurs. (If one is to interpret propositional quantification this way, then, for example, one will not be able to elucidate the meaning of the sentence "There are various propositions which are true" by such means, as according to the interpretation of propositional quantification being considered, propositions can only differ from each other in respect of their truth-value, and this, in turn, implies the following absurdity: that there are different truth-values identical with the truth.) So, if propositional quantification were to be intelligible, it would have to be understood in some other way. The only alternative to a referential interpretation seems to be a substitutional one. However, according to the above line of argumentation, this interpretation also does not allow one to adequately comprehend many general sentences about thoughts, beliefs and propositions.

The considerations presented so far show that the assumption (that propositional quantification must be either referential or substitutional) underlying the objection that propositional quantification does not allow one to explain the meaning of general sentences about thoughts, beliefs, and propositions is mistaken. Propositional quantification neither need be conceived of as referential, nor as substitutional (see Hugly and Sayward 1996, 241). To invoke Wittgenstein's *dictum*, the belief that there is such

a necessity stems from a one-sided diet. How, then, should propositional quantification be understood? In answering this question, I will limit myself to a few remarks. In my opinion, the key to a proper understanding of propositional quantification is the thought that sentences describe situations, but do not designate anything (Prior, 1971, 19; Wittgenstein 1922, 3.14, 3.1432, 3.144). So, propositional quantification is not used to form sentences about certain elements (e.g. propositions) which belong to a certain domain. The generality of a propositional variable does not consist in the fact that certain objects are its values, and the fact that these objects belong to a set which is the range of this variable. Its generality consists in the fact that it can be variously interpreted: that is, different senses can be assigned to it. However, it is worth noting that the latter explanation should not be understood as meaning that a propositional variable takes as its values objects called senses or thoughts. As Wittgenstein had already noted in the *Tractatus*, such a reifying conception of senses or thoughts is confused (1922, 3.144). So, if one assumes that a certain propositional variable, e.g. "*p*," can be interpreted in finitely many ways, then one can explain its generality as follows: the propositional variable "*p*" can be construed as saying that grass is green, that snow is white, and so on (where the expression "and so on" should be substituted with a finite list of sentences, each preceded by the expression "that"). Of course, if we assume that propositional variables can be interpreted in infinitely many ways, then it will not be possible for these interpretations to be given by enumeration; they can be furnished, *inter alia*, by means of a description, or by providing a finite set of the simplest interpretations, along with rules of formation for more complex ones. These remarks show that propositional quantification need not and should not be conceived as either referential or substitutional quantification.

In this regard, it is worth adding that an abandonment of the substitutional interpretation of propositional quantification need not mean that one cannot, in one's explanations of the meaning of general sentences about thoughts, beliefs and propositions, appeal to what one can substitute for propositional variables to obtain true sentences. For example, one can partly explain the meaning of the sentence "A certain thought of John is true" by pointing out that if one substitutes the sentence "Biden is older than Obama" for the propositional variable which occurs in the paraphrase of the sentence "A certain thought of John is true," one will obtain a true sentence provided that it is true that Biden is older than Obama and it is true that John thinks that Biden is older than Obama. Abandoning the substitutional interpretation of propositional quantification only entails giving up on the belief that sentences in which propositional quantification occurs have

the same meaning as certain counterparts of them that are true in virtue of the fact that propositional variables can be substituted with certain sentences. Further to this, I believe that to better understand the above remark it is worth invoking an analogy. As Wittgenstein rightly observed, mental states cannot be reduced to behaviour, but it is not possible to explain the essence of mental states without an appeal to outer criteria—i.e. behaviour. It seems that in the case of propositional quantification the situation is similar: the role of propositional variables cannot be reduced to the role of the expressions for which one can substitute sentences belonging to a given list or language, yet this role is intelligible only provided that there are examples of expressions which can be substituted for such variables.

As I have already mentioned, one of the reasons for questioning the intelligibility of propositional quantification is the belief that there are no expressions in natural languages that play a similar role to that played by propositional variables in artificial languages. In my view, this belief is mistaken because expressions such as "Things are thus and so," "This is how things are," "Things are this way," and "Such and such is the case" play a very similar role to propositional variables (Wittgenstein 2009, § 134; Prior 1971, 37–8; Hugly and Sayward 1996, 251). Moreover, there are sentences in natural languages that correspond to formulas in formal languages containing propositional quantification. "Things are somehow," "It is not the case that however things are or are not, it is true that they are thus and so" are examples of such sentences. (Alternative, but reifying, formulation of the second of these could be "Not all propositions are true.") A quasi-formal counterpart of the first sentence is "There is *p*, such that *p*," and a quasi-formal counterpart of the second one is "It is not the case that for every *p*, it is true that *p*."[13]

Let us consider some other examples of sentences in a natural language containing constructions that correspond to propositional quantification:

13. One may wonder why I use the expression "however things are or are not," instead of the shorter expression "however things are," in the second sentence. The reason for this is that if one used the shorter expression, the whole sentence would not have the intended meaning: namely, that not all propositions are true. Instead, it would mean that it is not always the case that if things are thus and so, it is true that they are thus and so. (The latter statement could be rendered in a quasi-formal notation in the following way "It is not the case that for every *p*, if *p*, it is true that *p*"). So, if one wants to express the universal propositional quantifier "for every *p*" in a natural language, one should use the longer phrase, because the shorter one, "however things are" corresponds to the following expression in quasi-formal notation "for every *p*, if *p*," whereas it is the longer phrase, corresponding to "for every *p*, if *p* or not-*p*," that truly captures what is meant here by "for every *p*" (the proposition "for every *p*, if *p* or not-*p*, then *p*" is logically equivalent to the proposition "for every *p*, *p*").

A. If, with regard to things being somehow, John believes that they are just thus and so and asserts that they are not just thus and so, then John is lying.
B. However things are or are not, if Eva knows that they are thus and so, then it is true that they are thus and so.

These sentences can be paraphrased in the following way:

A'. If for some *p*, John believes that *p* and asserts that not-*p*, then John is lying.
B'. For every *p*, if Eva knows that *p*, then it is true that *p*.

The fact that paraphrases of this kind seem adequate shows that, contrary to the criticism of propositional quantification presented above, there are counterparts of propositional variables in natural languages. Here, I would like to point out that the possibility of translating certain simple formulas involving propositional variables into natural-language sentences of the kind wherein unquestionable counterparts of propositional variables occur does not entail that such a translation would also be recognized as adequate in cases where we encounter more complex formulas with propositional variables. As it seems, a translation of more complex formulas into sentences of the sort wherein unquestionable counterparts of propositional variables occur could prove highly artificial, and the results of the translation would perhaps not even be recognized as forming correct sentences within some given natural language. For this reason, in natural languages there are virtually no very complex sentences composed of such counterparts of propositional variables that count—from the point of view of grammar in the linguistic sense of the term—as expressions playing the role of propositional variables. Indeed, the expressions commonly used in natural languages as counterparts of propositional variables are pronouns (see Prior 1971, 37–8), not prosentences, so according to many philosophers they do not play the role of propositional variables, but that of variable names (Horwich 1990, 4). However, in my opinion, this argument is misleading. First of all, what is essential is not the linguistically understood grammatical category of an expression, but its use. Secondly, it is easy to explain why such a reifying way of speaking is so pervasive in natural languages. This manner of speaking is simply much shorter than that which would express more perspicuously the grammar—in Wittgenstein's sense—of general sentences about propositions.

8. The Use of the Word "That"

As I have already indicated, another question to be considered in connection with the proposed elucidation of the use of truth-expressions in sentences 1–7 is the following: what role does the word "that" play in 1–5 and their paraphrases? In other words, we need to consider what the grammar—in Wittgenstein's sense—of the word "that" is in such contexts. One way to describe the grammar of a given expression in a given context is to offer an adequate paraphrase of that context. Below, I focus on three examples: sentences 2, 3 and 4. Before I present such paraphrases, I wish to remind the reader that according to an alternative approach to the one proposed above, the word "that" and the sentence that follows it form an expression that refers either to a proposition expressed by this sentence (in a given context) or to a fact described by it (see Parsons 1993). On the approach put forward in this text, on the other hand, the word "that" should rather be treated as naturally connected with the words "it is true" than with the sentence following this word, and such a whole should be interpreted as the truth connective "it is true that."

At this juncture, I should emphasize that sentences in which the expression "it is true that" occurs can be paraphrased in such a way that the word "that" does not occur in these paraphrases. Sentences 2, 3 and 4 can be paraphrased in the following ways:

2.' It is true: Kant read Hume.
2.'' The truth is: Kant read Hume.
3.' It is true: the Earth revolves around the Sun.
3.'' The truth is: the Earth revolves around the Sun.
4.' Ann asserts truly: Glenn Gould recorded the Goldberg Variations in 1981.

The word "that" does not occur in the paraphrases; it is substituted with a punctuation mark—namely, a colon. This shows that it is not necessary to use the word "that" in order to express the thoughts conveyed by 2, 3 and 4. What is essential, however, is the fact that on the basis of these paraphrases of 2, 3 and 4 one can see even more clearly that the grammatical unit (in Wittgenstein's sense of "grammar") in a sentence such as "It is true that Kant read Hume" is the sentence "Kant read Hume," not the expression "that Kant read Hume," which is assumed to designate the proposition expressed by the latter sentence. In the case of 2', the sentence "It is true: Kant read Hume," it is obvious that a logico-grammatical analysis that picked out the expression ": Kant read Hume" as a grammatical unit would be absurd.

To sum up these sketchy remarks concerning the role played by the word "that" in sentences in which truth-expressions occur, and the possibility

of paraphrasing such sentences, I wish to note that the expressions "the truth is:" and "it is true:," which occur in sentences such as 2', 2'', 3' and 3'', play the role of a sentence-forming functor that has a sentence as its argument. That this is so would seem to confirm the idea that construing the truth-expressions occurring in sentences 2 and 3 as performing the role of an operator is not only natural, but justified.

9. The Predicative Use of Truth-Expressions vs. Their Use as an Operator

As I have stated already, my view is that the truth-expression which occurs in sentences 6 and 7 ("The sentence 'London is not always foggy' is true" and "The first sentence of Frege's paper *On Concept and Object* is true") plays a different role from those truth-expressions that occur in 1–5: I think it should be treated as a genuine predicate. This assertion requires some elaboration, so I would like to briefly subject the issues flagged earlier to analysis. Firstly, I will try to answer the question of why this truth-expression should be treated as a predicate. Secondly, I wish to make explicit the meaning of this predicate. Thirdly, I will try to answer the following question: of what kind of objects is this predicate predicated? These three issues are interconnected, so I will not be dealing with them separately. The truth-expression that occurs in sentences 6 and 7 is a predicate because it is predicated of genuine objects: namely, sentences (see Prior 1971, 98). However, it should be emphasized that one does not use sentences 6 and 7 to make statements ascribing the alleged property of being true to the sentences mentioned in these sentences; 6 and 7 are rather employed to ascribe the genuine property of asserting (expressing) the truth to them. That is, according to the analysis proposed in this text, saying that a sentence is true (i.e. has the property of being true) is usually, in fact, an abbreviated way of saying that the sentence asserts the truth. Moreover, it seems that the possession of this property by sentences of a natural language is relative to context, and is so not only in the case of sentences of the kind in which occasion-sensitive expressions occur (see Travis 2000, 141–4). Let us consider the sentence "There are two apples on table X at time t," and a situation in which there are two significantly bitten apples on table X at time t (where "X" and "t" are constants). In certain contexts of evaluation (e.g., when assigning what is on the table to a particular taxonomic category of fruit) this sentence asserts (or expresses) the truth, while in other contexts of evaluation (e.g., when someone is asking whether there are any more apples to eat on the table) it does not. Of course, this last remark is only a digression, as the adequacy of the analysis of the role played by truth-expressions in our

language proposed above by no means depends on the legitimacy of contextualism (in the sense of the position according to which one and the same unambiguous sentence, involving no occasion-sensitive expressions, can be used in different contexts to make different statements).

To sum up, then, in sentences such as "The sentence 'London is not always foggy' is true" and "The first sentence of Frege's paper *On Concept and Object* is true" we encounter a predicate that is predicated of sentences, and its sense is more or less this: the sentence … asserts the truth.

In my view, what has been said so far about the role played by truth-expressions in such sentences as 1–7 allows one to elucidate the difference between Tarski's *Convention T* (see Tarski 1944) and the following schema:

TP It is true that *p* iff *p*.

The above considerations can also help us see why the problem of truth-bearers is, in fact, not a real problem, but rather either a question resulting from a conceptual confusion or one which has a quite trivial answer.

Taking into account the foregoing remarks on the role played by truth-expressions in sentences of type 6 and type 7, it would seem that *Convention T* ought to be reformulated in the following terms:

T' The sentence *s* asserts (expresses) the truth in a language *L* iff *p*. (Where the variable "*s*" should be substituted with the name of that sentence whose translation into the metalanguage is to be substituted for the variable "*p*.")

Although *Convention T*, and its reformulated version *Convention T'*, are schemas for intuitively true propositions, they are not laws of logic. Why not? To put the matter briefly, the reason is that predicates of the type "asserts truth in language *a*" (where "*a*" is the name of some specific language) are not logical constants. This claim can also be justified by the fact that the truth of such equivalences depends on that which is asserted (or expressed) by the sentence of which the property of being true (as in the original version of the *Convention T*) or the property of asserting (or expressing) truth (as in *Convention T'*) is predicated. The fact that a certain sentence in a given language asserts that things are thus and so and asserts the truth is not logically equivalent to the fact that things are thus and so. For example, that the sentence "Kant read Hume" asserts (or expresses) in English that Kant read Hume, and that this sentence asserts the truth, is not logically equivalent to this: Kant read Hume. Likewise, from the latter

it does not follow that the sentence "Kant read Hume" asserts (or expresses) in English that Kant read Hume and that this sentence asserts the truth.

Things look different in the case of the schema TP. This is a logical truth, because the only constant in this scheme is the expression "it is true that"—for which, as symbolic counterpart, we may use the sign "T"—which should be recognized as a logical constant. This expression plays the role of an unary logical connective, the meaning of which can be explicated by two rules:

1. The elimination rule for T

$$\frac{T\alpha}{\alpha}$$

2. The introduction rule for T

$$\frac{\alpha}{T\alpha}$$

If this connective is recognized as expressing the content of our ordinary concept of truth, then its meaning cannot be defined by means of a truth-table; however, it can—in some sense—be explicated by means of the following truth-table.

α	$T\alpha$
1	1
0	0

This truth-table is not a definition of the expression "it is true that," because the above explication of the meaning of this expression contains the symbol "1," which is commonly treated as designating the True. This fact can be interpreted in two ways: either as undermining the approach to the concept of truth developed in the present article, or as confirming that truth is in some essential sense indefinable. One can argue for the first option as follows. Since truth-tables are commonly recognized as correct definitions of (classically interpreted) logical connectives, if the expression "it is true that" cannot be defined by means of a truth-table, then it must be the case that truth-expressions do not play the role of connectives (i.e. operators)—or, at least, that they also play some other, more fundamental role

than that of an operator: e.g., that of a name or a predicate. As an argument for the second option, one can point out that the impossibility of defining the expression "it is true that" by means of a truth-table does not show at all that we do not understand the functioning of this expression as a connective. We use this expression as a connective without any problem, and this fact proves that we understand such a use of it well enough. So, the impossibility of defining the expression "it is true that" by means of a truth-table only implies that it is, in some sense, indefinable,[14] and not that it does not play the role of a connective (i.e. operator). It should be added that this way of understanding the truth-table for the connective "T," according to which it is not a definition of this connective, presupposes that the symbols "1" and "0" are not used as names of certain objects, but rather as incomplete symbols or *syncategorematic* expressions. It seems that Wittgenstein interpreted the use of "1" and "0" in truth-tables in just that way (see 1922, 4.441).

10. Conclusion: Dissolving the Problem of Truth-Bearers

As I have already stated, my view is that the above elucidation of certain aspects of the grammar (in Wittgenstein's sense) of truth-expressions allows one to dissolve the problem of truth-bearers by showing that it is either a problem resulting from a conceptual confusion or one with a trivial solution. In contexts where truth-expressions play the role of an operator, and not a predicate, they do not serve to ascribe a certain property to a certain object. Hence, in such contexts, there are no objects to which the property of being true is ascribed, as expressions such as "it is true that" do not play the role of a predicate and thus do not refer to any property such as would be allegedly possessed by objects of a certain kind. Also, it still needs to be emphasized that this observation is not an expression of some kind of metaphysical anti-realism with regard to truth, but rather a grammatical remark. Meanwhile, in cases where truth-expressions are properly interpreted as predicates with the form "the sentence … expresses (asserts) the truth in language *L*," the solution to the problem of truth-bearers is quite trivial. It is obvious that the properties to which predicates of this kind refer are properties of sentences: this fact is implied by the very meaning

14. It is worth emphasizing that according to some authors, indefinability is an important feature of the concept of truth (see Frege 1984b; Davidson 1996). Wittgenstein's remarks on truth in *Philosophical Investigations* are interpreted in a similar way by Bronzo and Vision (Bronzo 2019; Vision 2005). Moreover, such a view about truth can be supported by an account of truth-expressions that treats them as playing the role of operators, not predicates (see Salis 2019a).

of such predicates, as they just have the form "the sentence … expresses (asserts) the truth in language *L*." In other words, grammatical investigations do indeed indicate a solution to the problem at hand: the bearers of the properties to which predicates of the form "the sentence … expresses (asserts) the truth in language *L*" refer are, of course, sentences.

This way of treating the problem of truth-bearers is, in my opinion, in line with Wittgenstein's approach to philosophical problems. On the one hand, the philosophical question "What is a bearer of truth?" is not invested with any determinate sense in many contexts, because truth-expressions play the role of operators, not predicates, in the latter, and this is why posing the question leads to "the discovery of some piece of plain nonsense" (Wittgenstein 2009, § 119)—the nonsense that results from trying to answer that question. On the other hand, in other contexts the answer to the question is obvious and trivial. When one uses truth-expressions to mean more or less the same as what predicates of the form "the sentence … expresses (asserts) the truth in the language *L*" mean, the thesis claiming that the properties to which predicates of this kind refer are properties of sentences will be indisputable, and obvious to everyone. As he himself put it: "If someone were to advance *theses* in philosophy, it would never be possible to debate them, because everyone would agree to them" (Wittgenstein 2009, § 128).

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