



Tomasz Knopik

<https://orcid.org/0000-0001-5253-7545>

Maria Curie-Skłodowska University, Lublin, Poland,

tomasz.knopik@umcs.eu

Urszula Oszwa

<https://orcid.org/0000-0002-0300-909X>

Maria Curie-Skłodowska University, Lublin, Poland,

urszula.oszwa@mail.umcs.pl

Self-Determination in Maths Education: How to Strengthen Students' Positive Attitude to Mathematics and Develop Their Emotional/Social Competence (pp. 185–210)

Abstract

The aim of this study was to verify the effectiveness of the self-determination theory (SDT) in mathematics teaching methodology. In the experimental group, 62 fifth-graders had 10 months of maths lessons according to an original programme that prioritises three needs: autonomy, competence and relatedness.

The following research questions were formulated:

1. How did students' attitudes towards mathematics change under the influence of methodological interventions aimed at satisfying the three SDT needs?
2. Did the methodological solutions contribute to better emotional/social competences in the students?
3. Did the intervention result in higher maths achievement, measured by a maths knowledge and skills test and grades in the subject?

The findings included statistically significant differences in 1) positive attitude towards mathematics (measured by the semantic differential technique),

2) emotional/social competences (measured by the KA scale from the tool called TROS-KA) and 3) mathematical achievements (measured by a maths test and grades in school) in comparison to the control group (N = 59), where less positive attitudes towards mathematics, lower maths grades and no significant increase in emotional/social competences were observed.

Keywords: SDT, mathematical education, attitude towards mathematics, emotional/social competence, mathematical achievements

Introduction

The self-determination theory (SDT) is a meta-theory (Deci & Ryan, 1985, 2000) which presents the relation between satisfying three universal needs – autonomy, competence and relatedness (Deci & Ryan, 2002) – and an increase in intrinsic motivation (Deci et al., 1991), which further leads to contentment, satisfaction, fulfilment and well-being in a given field (Chirkov et al., 2003; Cuevas et al., 2018). So far, the SDT has been empirically confirmed by research in various areas: work efficiency, sport, medicine, psychotherapy and religion (Grolnick et al., 1997; Magne & Deci, 2005). Studies have also been carried out regarding its use in education (Trenshaw et al., 2016; Vansteenkiste et al., 2006; Niemiec & Ryan, 2009). The results confirm improvements in many individual variables under the influence of interactions targeting the three universal needs (Deci & Ryan, 2000).

According to the SDT a social environment conducive to the students' optimal development should be dominated by teachers' interactions and strategies supporting their intrinsic motivation (Deci & Ryan, 2002). As a result the intrinsically motivated students achieve higher grades in school than their externally motivated peers (Noels et al., 2000; Oga-Baldwin et al., 2017). They show higher self-esteem (Orsini et al., 2018) and they better understand the essence of the knowledge they acquire (Magne & Deci, 2005). They also reveal higher social competences (Reeve, 2002). They show better adaptability to changes and new situations (Niemiec & Ryan, 2009) and present higher self-awareness and emotional competence (Grolnick et al., 1997).

The SDT was developed in the 1980s by Ryan and Deci (1985), and it has been empirically confirmed at different levels of education (Cuevas et al., 2018; Kaplan, 2018). Its universal assumptions about optimal learning environments can be applied to specific school subjects and types of competences developed at school. Therefore, taking this as a starting point, supported by a review of the literature on the SDT in mathematics teaching (Hagger et al., 2015; Kiemer et al., 2015; Leroy & Bressoux, 2016), the authors assumed that the use of this concept in teaching methodology can contribute to more effective mathematics education, changes in students' attitudes towards mathematics and better emotional/social competences.

The authors' interest was directed to the application of the SDT's assumptions in teaching mathematics for two reasons. Firstly, the changing nature of life in the digital technology era necessitates a redefinition of the learning process and its goals. One can notice a gradual transition from teaching towards learning with an emphasis on student activity in this process (student-centred learning [Justice et al., 2009]). Secondly, despite numerous reports on the increase of intrinsic motivation to learn mathematics, there has not been much research on using the SDT to change attitudes towards maths. This is worth paying special attention to because mathematics is more often burdened with negative emotions than other school subjects (Carmichael et al., 2017). There are several reasons for this. Firstly, mathematics has the most abstract character among fields of knowledge. Its content is taught spirally in schools, which means that its understanding at higher levels depends on prior education (Brandenberger et al., 2018). Therefore, a lack of comprehension of the basics of mathematics in the early school period can lead to further difficulties in subsequent years of education (Corkin et al., 2018). Secondly, mathematics is about solving problems, and this requires, among other skills, the ability to deal with difficulties (Kiemer et al., 2015). In the student's mind, getting the correct solution to a maths problem is a type of reward, while incorrect solutions lead to frustration and poor grades (Bourgeois & Boberg, 2016; Brandenberger et al., 2018).

The SDT in mathematics education – A research review

The ability of self-determination (Niemi & Ryan, 2009) and engaging in activities driven by interest and joy is an important factor in increasing the effectiveness of learning and teaching (Vansteenkiste et al., 2006). It seems particularly important to equip primary education students with this skill so that they have proper, durable patterns for acquiring and constructing knowledge in their future. According to the authors of the SDT, satisfying the need for autonomy, competence and relatedness is the crucial condition for developing intrinsic motivation (Deci & Ryan, 1985, 2002). This is particularly relevant in maths education due to the waning interest in the subject in later educational stages (Carmichael et al., 2017; Lohbeck, 2018) and the build-up of negative emotions around it (Leroy & Bressoux, 2016; Liu et al., 2016; Humberg, 1990; Ma & Xu, 2004; Carey et al., 2016). Studies based on the SDT in maths education relate to 1) environmental conditions of intrinsic motivation to learn mathematics, 2) emotions accompanying learning mathematics and 3) primary school mathematics teaching as the basis for further mathematics education.

Environmental conditions of intrinsic motivation to learn mathematics

The participation of environmental factors in the process of learning is well reflected in the idea of 'smart context' (Barab & Plucker, 2002), which consists in creating didactic situations to enhance students' potential. For a more precise explanation of this concept, one can look to ecological psychology (Gibson, 2000). From a relational perspective (Barab & Roth, 2006), an action can only occur if a person recognises environmental features that provide opportunities for the action. Two terms define this relationship accurately: *affordance* and *effectivity* (Gibson, 2000). *Affordances* relate to the possibilities of action provided by the environment (Gibson, 2000; Young et al., 2002), while *effectivity* is the behaviour that an individual can generate in response to these possibilities (Barab & Roth, 2006). With low *affordance*, it is difficult for a person to be

effective. In early education, this is difficult because the student's limited learning experience cannot yet provide them with the knowledge necessary to take the initiative to engage in an activity: the subject is not yet aware that they are able to do it (Gibson, 2000).

Papanastasiou (2008) sought the indicators of the school environment that determine students' achievement in maths. Data was collected from 3,116 eighth-graders (mean age: 13.8 years) participating in the Trends in International Mathematics and Science Study, which constituted about 31.8% of the total student population of this age in Cyprus (9,786). By analysing the differences between the expected and achieved results, the researchers identified schools that perform better than might be expected. The analysis revealed six factors that explained the differences associated with mathematics achievement. Transmission teaching was the factor that explained most of the difference between more and less effective schools. The others were active learning, self-perception – including maths self-concept – students' attitude to maths, family incentives and classroom atmosphere (Papanastasiou, 2008). The study showed that environmental factors play a crucial role in developing students' intrinsic motivation to learning.

Davadas and Lay (2018) indicated that students' approach to maths was influenced by their parents' attitudes and their teachers' emotional support and teaching instructions. The study analysed the relationship between these factors and students' attitude towards mathematics. The sample of 318 Malaysian fourth-graders was assessed with a questionnaire measuring a) parents' attitude towards mathematics, b) support from mathematics teachers, c) the way mathematics was taught in the classroom and d) the students' attitude towards mathematics. It was found that teacher support and teaching method had the most significant impact on the students' positive attitude towards mathematics (Davadas & Lay, 2018).

The quasi-experimental study by Brandenberger et al. (2018) consisted in a one-year classroom multicomponent intervention with 348 Swiss seventh-graders, divided into three groups: 1) student/teacher intervention group, 2) student intervention group and 3) no intervention (control) group. Intrinsic and identified motivation were self-reported by the students.

The results showed significantly less of both types of motivation in the student intervention group. The most positive effects were found in the student/teacher intervention group, which the authors interpreted as value-constructed successful learning.

These studies show the role of the environment – the teachers' participation, in particular – in developing intrinsic motivation to learn mathematics. As a consequence, it should lead to higher maths achievement.

Motivation and emotions in maths education and students' maths achievement

A longitudinal study by Leroy and Bressoux (2016) among 1,082 French sixth-graders analysed the relationship between six types of SDT motivation and maths achievement during the first year of junior high school. Maths teaching based on the SDT led to higher intrinsic motivation and less amotivation after a one-year intervention. Similar results were obtained in a study by Kiemer et al. (2015), which sought a strategy to combat the decline in student interest and motivation for maths learning throughout secondary education in Germany. Teaching based on the SDT and respecting the students' universal needs was a positive remedy for this decline in the experimental group compared to the control group, where the methods were not applied. A study of 216 Pakistani students aged 12–15 years (Hagger et al., 2015) assessed the processes of students' perceived autonomy support and autonomous forms of motivation towards mathematics in the classroom, and indicated them as predictors of autonomous motivation toward maths homework achievement. Perceived autonomy support was defined as students' perception that their teachers support their intrinsic motivation. Autonomous forms of motivation were defined as a sense of choice, ownership and self-efficacy. The results provided evidence that students' autonomous motivation towards maths at school, supported by teachers, is strongly linked to maths interest in doing and engaging in homework.

A longitudinal study based on the SDT and the control value theory (CVT), conducted by Sutter-Brandenberger et al. (2018), concerned the relationship between self-determined motivation and three negative

emotions in learning mathematics: a) anxiety, b) anger and c) boredom. To assess emotions and motivation among 348 seventh-graders, three self-report measures were made: 1) at the beginning of the seventh grade, 2) at the end of the seventh grade and 3) at the end of the eighth grade. The results demonstrated unidirectional negative effects between self-determined motivation and the three tested emotions. This means that the increase in students' intrinsic motivation in learning maths caused lower levels of anxiety, anger and boredom associated with the subject.

Primary school mathematics teaching as the basis of further mathematical education

A study by Lohbeck (2018) measured 397 children at the beginning of education (mean age: 9.55 years) in terms of maths self-concept and the SDT's six types of motivation: intrinsic, integrated, identified, introjected, extrinsic motivation and amotivation (Deci & Ryan, 1985; Vallerand, 2001). The relationships between maths self-concept, motivation and grades in maths were explored. Statistically significant positive correlations between maths self-concept, intrinsic, integrated and identified motivation and maths grades were obtained. The strongest connection was observed between maths self-concept and maths grades.

Numerous observations of the decline in intrinsic motivation to study mathematics and engagement in later schooling led researchers to search for the causes of this issue. A study by Bourgeois and Boberg (2016) investigated the reasons for the decrease in intrinsic motivation and involvement in school life among high achieving maths students (grades 3 to 8). It was conducted on a huge sample of students (N = 5,392) and teachers (N = 680) in the southern USA. A mixed methodology was used: self-reports, classroom observations, interviews with maths teachers/principals/students and surveys from parents. The analysis was arranged in three pathways related to the SDT's universal needs: autonomy, competence and relatedness. The results indicate that high achieving students who showed relatively low scores on school life engagement shared nine similarities: 1) low connection to school, 2) low school enjoyment, 3) negative or neutral attitude towards other school subjects, 4) optimal

challenge preferences, 5) enjoyment of mathematics homework, 6) competitiveness, 7) attitude towards grades, 8) parental involvement and 9) extrinsic incentives. In the context of decreasing involvement and intrinsic motivation with age and length of education, it seems important to ensure that students are equipped with this type of motivation from the youngest primary school grades. Maths self-concept, which has the greatest correlation with the results in maths learning, also required similar strengthening (Bourgeois & Boberg, 2016).

Based on this recent research, the decision was made for the present study to measure the attitudes towards mathematics among 62 fifth-grade students who underwent (for 10 months, the whole school year) an original programme based on the SDT, aimed at supporting and developing the three universal needs – autonomy, competence and relatedness – as well as intrinsic motivation to learn mathematics.

The following research questions were formulated:

1. How did students' attitudes towards mathematics change under the influence of methodological interventions aimed at satisfying the three SDT needs?
2. Did the methodological solutions contribute to better emotional/social competences in the students?
3. Did the intervention result in higher maths achievement, measured by a maths knowledge and skills test and grades in the subject?

Study design

The substantive and methodological basis of the study was the original model of measuring emotional/social competences (Domagała-Zyśk et al., 2017). This model was based on the intelligence concept conducive to success in life by Sternberg (2003) and on Erikson's (2004) psychosocial development theory. The authors defined the general sense of competence as a set of one's emotional and social skills: coping with difficulties (T scale), social relations (R scale), self-concept (O scale), sense

of agency (S scale) and affect control (KA scale). The KA scale, created from the strongest items of the TROS scales, is a screening tool for general assessment of emotional/social competences. These skills can be treated as transferable resources (Pellegrino, 2012), i.e. skills necessary for tasks both of a typically school nature and those related to everyday life. From the letters defining the individual competences in Polish, the acronym TROS-KA was created for the name of the instrument.

Method

Participants

The study was conducted according to the test-retest model using the semantic differential technique, a mathematics achievement test and the TROS-KA test tool, in September 2017 and June 2018 (after the entire school year) among six fifth-grade students (Polish primary school consists of eight grades) in three primary schools. In total, 121 students (including 64 girls and 57 boys) aged 10–11 years were included in the study, 62 of which had the intervention and 59 of which constituted a control group, where the classes were carried out according to the traditional programme and methodology.

All the fifth-graders at the schools participated in the study. Care was taken to ensure that during the two meetings and individual consultations, the parents were thoroughly informed about the objectives and course of the experiment. Their consent for the assessment of their children and the appropriate consent of the local research ethics committee were obtained. In addition, consultations were held with mathematics education methodologists to minimise the risk associated with the students' participation in the study. For students who did not agree to take part in the experiment, it was possible to attend mathematics classes in a parallel grade with the traditional curriculum; no student made use of this opportunity. The research was carried out in schools whose principals responded to the request of the researchers.

Measurements and procedure

The longitudinal study was run in three stages:

- 1) initial measurement of the current development of students' emotional/social competencies (ESC), attitudes towards mathematics and mathematics achievement,
- 2) conducting mathematics teaching based on SDT assumptions and
- 3) final measurement of students' ESC, their attitudes towards mathematics and their mathematics achievement.

1. Assessment of the current development of students' ESC, attitudes towards mathematics and mathematics achievement

During the one-hour meeting in groups of up to five students, their development of ESC was measured with the use of the TROS-KA tool (KA scale). In addition, the attitude towards mathematics was measured using the semantic differential technique, which is a combination of two methods: scaling and associative (Czapiński, 1978). The assumption of the method is that concepts can be defined in several dimensions so that a specific semantic space is created. The authors, Osgood et al. (1957), assumed that this semantic space could serve as an accurate and reliable tool for studying a person's attitude. The study used a semantic differential technique with a modified list of adjectives, based on Czapiński's (1978) work. The subjects' task was to evaluate each of 10 pairs of opposing adjectives by referring to mathematics (the measure of attitude). The highest numerical value (7) was assigned to objectively positive connotative adjectives, while the lowest (1) was assigned to objectively negative connotative ones. The use of the semantic differential technique allowed the researchers to investigate the students' current affective and cognitive assessment of mathematics (understood mainly as a school subject). For measurement of mathematical achievements we used maths knowledge and skills test based on the basis of the contents of the core curriculum for the fifth grade – up to a maximum of 50 points.

2. Conducting maths teaching based on SDT assumptions

Due to the fact that the most important stage of the study was implementing an appropriate – according to the assumptions of SDT – methodological workshop that differs greatly from the one used so far, the procedure included a 34-hour training programme and consultations with experts. Additionally, teachers were offered assistance in the form of mentoring at every stage of the research. During the training sessions, the teachers independently constructed the scenarios of the classes, evaluated them collectively and introduced modifications under the supervision of an expert. Three maths teachers (with similar professional experience, qualifications and from 12 to 15 years of work experience) taught maths based on the assumptions resulting from the training for 10 months (see Tables 1 and 2 for details). The research included monitoring of didactic activities: two didactics experts visited the classes at least once a month.

Table 1. Examples of maths teaching strategies based on the SDT and used in the study

SDT area of intervention	Methodological solutions
Relatedness	1. With the help of the teacher, teams of students set monthly goals to be achieved in mathematics. These goals were visualised in different parts of the room. The students felt responsible for the learning outcomes of their classmates and implemented elements of peer tutoring. 2. During the school year, the students completed three projects for their maths lessons. Each project was divided into parts, which were assigned to different teams. The project presentation stage required work consolidation and cooperation between all teams. The topics were: – <i>Butterfly effect in saving the planet</i> – <i>Logics of algorithms</i> – <i>Alternative counting systems</i> 3. The motivating thoughts of famous figures from the world of art, science and sport were chosen by the class as the motto of the week. 4. Class cheers encouraged joint effort, especially when working on difficult material.
Competence	1. In introducing and summarising lessons, the teacher pointed out the practical possibilities of using the analysed mathematical content (e.g. in technology, IT, individual saving plans, etc.). After familiarising themselves with the topic, students pointed to additional applications. 2. The operational goals from the core curriculum in maths were treated as an opportunity to exceed one's own limits. 3. A maths advertising campaign was conducted: once a week, students created posters and advertising slogans to encourage each other to master a given mathematics issue.

Autonomy	<ol style="list-style-type: none"> 1. Multilevel tasks – common task content with a dozen commands that the student chooses themselves (depending on their abilities and individual pace of work) 2. Possibility of students deciding together about the method of practicing, choosing exercises, work cards, etc. 3. Alternative ways to check knowledge – the student chooses from at least three different forms 4. Possibility of temporarily non-engaging, allowing for rest, calming down or taking care of the student's chosen priority 5. A wide selection of optional content – students indicate additional issues that they think should be accomplished during the lesson 6. The teacher's ranking of methods and content of education (based on student surveys) provides an opportunity for students to express their own individual assessment. The results of the surveys were discussed regularly and were used to implement changes.
-----------------	--

Table 2. Description of SDT implementation in the context of maths curriculum

The aim of education in the maths core curriculum (fifth grade)	Detailed implementation methods based on the SDT
<i>The student reads with comprehension text containing numerical information.</i>	<ol style="list-style-type: none"> 1. Students collect excerpts of online newspaper articles (print-outs) containing numerical information for two weeks (<i>student's independent activity, autonomy in the selection of information</i>). 2. During class, they write down examples of numerical information taken from the collected materials and form a matrix for playing bingo (<i>autonomous selection of the student</i>). They look for people who also wrote down the same numbers. The task for the class is to bring about a situation where every student will be able to say 'BINGO', i.e. all numbers will be paired (<i>relatedness instead of competition</i>). In case of difficulties, the students modify the rules, e.g. they look for the same or similar numbers (<i>autonomy, shared responsibility, sense of competence</i>). 3. The teacher poses a problem: How many different texts with numbers appeared today? Try to estimate. 4. Students make hypotheses and verify them by calculating the texts accurately. The teacher sums up the importance of numbers in providing information about the world (<i>need for competence</i>). As part of a creative summary, the students discuss in a circle: What can't be counted? 5. The teacher gives additional instructions: If someone remembers that an uncountable object mentioned in the discussion appeared in one of the previously read texts as countable, they say loudly: 'CHECKING' (<i>need for autonomy, competence and relatedness</i>). <p>Example: The student mentioned 'feeling' as an uncountable object. A classmate shouted 'CHECKING' and cited the text she read: 'In her life only two feelings mattered: love and hate.' Based on this example, the students discussed whether feelings are measurable or immeasurable (<i>need for autonomy and competence</i>).</p>

<p><i>The student correctly uses the units of volume and capacity.</i></p>	<ol style="list-style-type: none"> 1. Students work using mathematics research stations. 2. They form four teams according to their own classification (<i>sense of autonomy</i>). 3. Each team prepares a station, at which students from the other teams will have mathematical challenges related to measuring the volume and capacity of objects/figures. 4. Students come up with a name and marketing slogan for their station, and prepare a poster together (<i>need for relatedness</i>). 5. Tasks are solved in teams and the points are earned for the team, rather than the individual. 6. The teacher encourages the teams to come up with tasks related to an object: the Sahara Desert, the Baltic Sea or Mont Blanc. 7. This provokes problems: How do you count the grains of sand in the desert/a sandbox/a bucket? 8. As part of the summary, students generate life situations in which units for capacity/volume are used (<i>need for competence</i>).
--	--

3. Final measurement of students' ESC, attitudes towards mathematics and mathematics achievement

After 10 months the development of students' ESC was again measured using the KA scale, and their attitude towards mathematics (semantic differential technique) and mathematical achievements (maths knowledge and skills test) Additionally, longitudinal data on students' maths achievement were obtained from the school a) at the end of the fourth grade, b) after the first semester of the fifth grade and c) at the end of the fifth grade.

Data analysis and results

The results regarding the attitude towards mathematics at the end of the fifth grade turned out to be higher in the experimental group (Table 3). In this group, the positive attitude increased between the two measurements ($M1 = 44.19$; $M2 = 51.39$), while in the control group the positive attitude decreased ($M1 = 42.85$; $M2 = 38.32$).

The differences between the experimental and control groups was significant in terms of both attitudes towards mathematics and ESC (Table 4). Students from the intervention group obtained significantly higher results on the KA scale, which indicated a higher level of affect control, defined in the TROS-KA model as a degree of emotional maturity allowing for effective functioning in a social group (Knopik & Oszwa, 2022).

Table 3. Experimental (E) and control (C) group results for attitude towards mathematics and emotional/social competences

Group		Differential1	Differential2	KA1	KA2
E	M	44.19	51.39	52.98	59.74
	N	62	62	62	62
	SD	13.180	12.877	10.912	10.457
C	M	42.85	38.32	53.51	54.10
	N	59	59	59	59
	SD	13.941	16.607	12.114	12.114

Table 4. Significance of differences in the final measurement, by group

Significance test of differences	Differential2	KA2
Mann–Whitney U	1027.000	1326.500
Wilcoxon W	2797.000	3096.500
Z	-4.161	-2.608
p	<0.001	0.009

It is worth noting that the observed differences between the groups in attitudes towards mathematics and level of ESC did not occur at the beginning of the study (the initial measurement in September; see Table 5). This supports the conclusion that the intervention contributed to these differences.

Table 5. Significance of differences in the initial measurement, by group

Significance test of differences	Differential1	KA1
Mann–Whitney U	1755.500	1751.500
Wilcoxon W	3525.500	3704.500
Z	-0.382	-0.402
p	0.703	0.687

The measurement of mathematical achievements shows that students from the experimental group at the end of the fifth grade obtained both better school grades and higher scores in the mathematical knowledge and skills test (Table 6). These differences were statistically significant (Table 7). The data in Table 6 show that the control students had slightly higher maths grades at the end of the fourth grade ($M = 4.07$) compared to the students from the experimental group ($M = 3.76$). These differences were not statistically significant, though it is worth using them to highlight the trend observed in both groups in the longitudinal perspective: a clear progression of improving maths grades in the experimental group (increasing average grades in subsequent semesters: 3.76, 3.89 and 4.42) and a clear regression in the control group (decreasing average grades in subsequent semesters: 4.07, 3.88 and 3.66).

Table 6. Mathematics achievement in the study groups

Group		Maths knowledge and skills test	Maths final grade in the fourth grade	Maths grade in the first term of the fifth grade	Maths final grade in fifth grade
E	M	39.90	3.76	3.89	4.42
	N	62	62	62	62
	SD	6.772	0.987	0.925	0.897
C	M	33.53	4.07	3.88	3.66
	N	59	59	59	59
	SD	9.730	0.980	0.892	0.993

Table 7. Significance of differences in maths test and grades (final measurement), by group

Significance test of differences	Maths knowledge and skills test	Maths final grade in the fourth grade	Maths grade in the first term of the fifth grade	Maths final grade in the fifth grade
Mann–Whitney U	1103.500	1476.500	1776.000	1138.000
Wilcoxon W	2873.500	3429.500	3729.000	2908.000
Z	-3.769	-1.924	-0.299	-3.772
p	$p < 0.001$	0.054	0.765	$p < 0.001$

The intra-group differences in the results between the initial and final measurements (Table 8) revealed a) significant increase in ESC in the experimental group and no such change in the control group, b) a significant increase in positive attitude towards mathematics among students from the experimental group alongside a significant decrease in this measurement in the control group and c) a significant increase in the maths achievement of students from the experimental group alongside a significant corresponding decrease in the control group.

Table 8. Significance of test-retest differences between the groups

Wilcoxon Rank Test		Maths grade first term of the fifth grade - Maths final grade in the fourth grade	Maths final grade in the fifth grade - Maths final grade in the fourth grade	Differential2 - Differential1	KA2 - KA1
E	Z	-2.309	-6.105	-6.411	-6.185
	p	0.021	p<0.001	p<0.001	p<0.001
C	Z	-2.294	-3.489	-3.933	-1.278
	p	0.022	p<0.001	p<0.001	0.201

Discussion

The research showed the effectiveness of methodological actions based on the assumptions of the SDT in a) the acquisition of mathematical knowledge and skills, b) forming a positive attitude towards mathematics and c) developing emotional and social competences. As shown by the examples of teacher activities in Table 4, the maths lessons were not limited only to transferring issues from the core curriculum, but instead were treated as a space conducive to the needs of students in three key aspects: autonomy, competence and relatedness (Deci & Ryan, 2002). In focus talks at the end of the study, the teachers directly admitted that they began to think more about their role as a mathematics teacher ('I realised that the concern for the well-being of students and the usefulness of mathematics in their lives after school is just as important as the transmission of

mathematics content'). The teachers' interventions with the experimental group were also focussed on developing ESC: cooperation, negotiation, dialogical thinking, dealing with frustration in the event of difficulties with a complicated problem, developing self-knowledge and stabilising self-esteem (see Table 3). It should be noted, however, that activities aimed at a) satisfying the three needs defined in the SDT, b) increasing intrinsic motivation and c) the development of ESC are not separate activities in practice (Justice et al., 2009). As Ryan and Deci (2000) stated, the implementation of SDT assumptions creates optimal conditions for developing ESC, acceptance and satisfaction of the following needs:

- a) autonomy – developing self-knowledge and forming self-esteem, a sense of agency and internal locus of control, responsibility for one's actions
- b) competence – identifying one's own resources, sense of meaning, task engagement, setting appropriate goals
- c) relatedness – the ability to cooperate, develop empathy, emotional control, cooperative learning, forming one's identity in a group, expressing one's opinion

This shows that, in fact, the use of the SDT with students contributes to the development of resources which can be successfully utilised in other contexts, including outside of school. This conclusion is particularly important in the scope of Polish education, in which there is strong opposition between implementing the core curriculum and developing the competence of transferable resources (Knopik & Oszwa, 2022; Stevens & Miretzky, 2014) without attempting to include both goals in the teaching methodology.

The significant decrease in the positive attitude towards mathematics in the control group is consistent with the standard phenomenon of negative attitudes towards mathematics growing among students in subsequent stages of education. However, the study suggests that this process can be stopped, and a positive attitude towards mathematics can be systematically built thanks to the methodological actions based on

the SDT's assumptions. The role of a wise and smart learning context should be recognised as important in this process (Gibson, 2000; Young et al., 2002). The examples listed in Table 1 show this relation: *the research station* triggers situations of mathematical investigation and experimentation instead of providing ready-made solutions, and *the bingo game* provides an opportunity to process numerical information at an intensity level that is difficult to achieve without the student's emotional involvement. This is in line with the results of Kunter et al. (2008), who stated that enthusiasm for mathematics was passed on to students, especially when teachers created a supportive social environment in the classroom and organised classes in the zone of proximal development of students (in accordance with the assumptions of the SDT: challenges adapted to the possibilities). Similar conclusions can be found in Blazar (2015) and Frenzel et al. (2009).

The intervention carried out during the maths lessons enabled the participants to develop ESC. A comparative analysis of the KA scale scores obtained by the students from the experimental and control groups confirms that the observed increase was not the result of natural developmental processes, but of planned educational interactions. Referring to the authoritative operationalisation of the ESC of the TROS-KA model, it is worth emphasising that the key aspects measured by the KA scale are a) dealing with difficulties, b) maturity in creating and maintaining social relations, c) self-concept and d) a sense of agency.

A previous study by Knopik and Oszwa (2019) showed that providing students with a learning environment that respects their needs – defined in the SDT concept as fundamental – contributes to the increase of ESC. Therefore, it seems reasonable to look at more positive attitudes towards mathematics through the prism of not only more intrinsic motivation to learn the subject, but also of improving the ESC necessary for effective maths learning, primarily the ability to cope with difficulties.

The proposed methodology of teaching mathematics is also effective in terms of learning the material. Students from the experimental group earned higher grades in mathematics and better results in the test of knowledge and mathematical skills than their peers from the control

group. To sum up, it can be concluded that involving a student in deciding about their education, combined with constant reference to everyday life and justification of activities allows them to activate additional motivation in case of difficulties (Armoura et al., 2015). Teachers implementing the programme in the experimental group agreed that the main change in students' behaviour was about more constructively coping with failures and treating them as challenges, not distractions. This readiness to work hard and carry on despite encountering problems seems to be crucial in learning mathematics. Importantly, it is transferable (Sawin, 2004; Shah, 2013) and it makes school more of a laboratory of competences useful in life (Stevens & Miretzky, 2014).

Conclusions

The study shows that the application of the SDT's assumptions in the methodology of mathematics teaching resulted in three parallel effects:

- 1) a very positive attitude towards mathematics,
- 2) stronger emotional/social competences of the students and
- 3) more effective mathematics learning, demonstrated in school achievement compared to the standard approach without the SDT.

This is due to a new, definitely different view of the didactic situation, which has not only been reduced to the formal occasion of transferring mathematical knowledge and implementing the core curriculum, but has also been treated as a complex relationship between the students' needs and the current learning context. Teaching that is based on the teacher's constant inclination to create an environment that is conducive to satisfying students' three universal needs (autonomy, competence and relatedness) allows students to take responsibility for the learning process and the associated increase in personal involvement (Van Roekel, 2001). This effect is particularly important in mathematics, considered by many older primary school students to be complicated and difficult to master.

It consolidates the syndrome of mathematical helplessness (Ciżkowicz, 2017) that constitutes a key barrier in the development of mathematical competence.

Furthermore, as the results show, the teaching methodology based on the SDT was also effective in supporting the development of students' ESC. The implementation of the SDT's assumptions in the philosophy of mathematics teaching, especially at an early stage of education, when the learner's attitudes towards subjects and their own resources are being shaped, allows realistic expectations for changing this negative paradigm in school. The SDT changes the systemic question of 'What does the student do to master the core curriculum?' to 'What challenges does the school create so that the student engages in the core curriculum?' Mathematics, therefore, from the obligation imposed by the system of learning abstraction, with which the student often does not feel an emotional connection, should become a set of development challenges tailored to the individual's abilities and needs, which one cannot remain indifferent to.

Limitations and further research

Due to the relatively small number of students participating in the study, great care should be taken in formulating general conclusions. The schools that took part in the experiment were open to testing new solutions and showed a strong desire to implement changes to existing educational practices. Less openness to novelty and the moderately conservative attitude of teachers that dominates Polish schools may constitute significant barriers in implementing the SDT's assumptions widely in educational practice.

Identifying components of school organisational culture as features of a learning environment conducive to the effective application of SDT principles in everyday teaching practice can be the next stage of study. It would also be interesting to continue the research over a longer period, e.g. in the case of the Polish education system, until the completion of primary school (variable measurements in grades 5–8). This would help

determine the results' sustainability and their potential development determinants.

In terms of application, the challenge is to interest mathematics teachers in using SDT in educational practice so that they begin to perceive enthusiasm for the mathematics of their students as a real goal, not a utopia doomed to failure.

References

- Armoura, C., Berjot, S., Gillet, N., Caruana, S., Cohen, J., & Finez, L. (2015). Autonomy-supportive and controlling styles of teaching: Opposite or distinct teaching styles? *Swiss Journal of Psychology, 74* (3), 141–158.
- Barab, S., & Plucker, J. A. (2002). Smart people or smart contexts? Cognition, ability, and talent development in an age of situated approaches to knowing and learning. *Educational Psychologist, 37*(3), 165–182.
- Barab, S., & Roth, W.-M. (2006). Curriculum-based ecosystems: Supporting knowing from an ecological perspective. *Educational Researcher, 35*(5). DOI: 10.3102/0013189X035005003
- Blazar, D. (2015). Grade assignments and the teacher pipeline: A low-cost lever to improve student achievement? *Educational Researcher, 44*(4), 213–227. DOI: 10.3102/0013189X15580944
- Bourgeois, S., & Boberg, J. (2016). High-achieving, cognitively disengaged middle level mathematics students: A self-determination theory perspective. *Research in Middle Level Education, 39*(9), 1–18. DOI: 10.1080/19404476.2016.1236230
- Brandenberger, C., Hagenauer, G., & Hascher, T. (2018). Promoting students' self-determined motivation in maths: Results of a one-year classroom intervention. *European Journal of Psychology of Education, 33*(2), 295–317. DOI: 10.1007/s10212-017-0336-y
- Carey, E., Hill, F., Devine, A., & Szűcs, D. (2016). The chicken or the egg? The direction of the relationship between mathematics anxiety and mathematics performance. *Frontiers in Psychology, 6*. DOI: 10.3389/fpsyg.2015.01987
- Carmichael, C., Muir, T., & Callingham, R. (2017). The impact of within-school autonomy on students' goal orientations and engagement with mathematics. *Mathematics Education Research Journal, 29*(2), 219–236.
- Chirkov, V., Ryan, R. M., Kim, Y., & Kaplan, U. (2003). Differentiating autonomy from individualism and independence: A self-determination theory perspective on internalization of cultural orientations and well-being. *Journal of Personality and Social Psychology, 84*(1), 97–110.
- Ciżkowicz, B. (2017). Wyuczona bezradność jako zmienna pośrednicząca między stylem atrybucyjnym a osiągnięciami akademickimi [Learned helplessness

- as a mediating variable between attributive style and academic achievement]. *Przeгляд Badań Edukacyjnych*, 24(1), 79–100.
- Corkin, D., Ekmekci, A., & Parr, R. (2018). The effects of the school-work environment on mathematics teachers' motivation for teaching: A self-determination theoretical perspective. *Australian Journal of Teacher Education*, 43(6), 50–66.
- Cuevas, R., Nitoumanis, N., & Fernandez-Bustos, J. (2018). Does teacher evaluation based on student performance predict motivation, well-being, and ill-being? *Journal of School Psychology*, 6(68), 154–162.
- Czapiński, J. (1978). Dyferencjał semantyczny [Semantic differential]. In L. Wołoszynowa (Ed.), *Materiały do nauczania psychologii* (seria III, t. 3) [Materials for teaching psychology, series III, Vol. 3] (pp. 241–256). PWN.
- Davadas, S., & Lay, Y. (2018). Factors affecting students' attitude toward mathematics: A structural equation modelling approach. *Eurasia Journal of Mathematics, Science and Technology Education*, 14(1), 517–529.
- Deci, E., & Ryan, R. (1985). *Intrinsic motivation and self-determination in human behavior*. Plenum.
- Deci, E., & Ryan, R. (2000). The “what” and “why” of goal pursuits: Human needs and the self-determination of behavior. *Psychological Inquiry*, 11, 227–268.
- Deci, E., & Ryan, R. (2002). *Handbook of self-determination research*. University of Rochester Press.
- Deci, E., Vallerand, R., Pelletier, L., & Ryan, R. (1991). Motivation and education: The self-determination perspective. *Educational Psychologist*, 26(3), 325–346.
- Domagała-Zyśk E., Knopik, T., & Oszwa, U. (2017). *Diagnoza funkcjonalna rozwoju społeczno-emocjonalnego uczniów w wieku 9-13 lat* [Functional diagnosis of socio-emotional development of students aged 9–13]. Ośrodek Rozwoju Edukacji.
- Erikson, E. (2004). *Tożsamość a cykl życia* [Identity and the life cycle]. Poznań: Wydawnictwo Zysk i S-ka.
- Frenzel, A., Goetz, T., Lüdtke, O., Pekrun, R., & Sutton, R. (2009). Emotional transmission in the classroom: Exploring the relationship between teacher and student enjoyment. *Journal of Educational Psychology*, 101(3), 705–716.
- Gibson, E. (2000). Where is the information for affordances? *Ecological Psychology*, 12, 53–56. DOI: 10.1207/S15326969ECO1201_5.

- Grolnick, W., Deci, E., & Ryan, R. (1997). Internalization within the family: The self-determination theory perspective. In J. Grusec & L. Kuczynski (Eds.), *Parenting and children's internalization of values: A handbook of contemporary theory* (pp. 135–161). John Wiley & Sons.
- Hagger, M., Sarwat, S., Hardcastle, S., & Chatzisarantis, N. (2015). Perceived autonomy support and autonomous motivation toward mathematics activities in educational and out-of-school contexts is related to mathematics homework behavior and attainment. *Contemporary Educational Psychology, 41*, 111–123.
- Humbree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education, 21*(1), 33–46.
- Justice, C., Rice, J., & Warry, W. (2009). Developing useful and transferable skills: Course design to prepare students for a life of learning. *International Journal for the Scholarship of Teaching and Learning, 3*, 2–9.
- Kaplan, H. (2018). Teachers' autonomy support, autonomy suppression and conditional negative regard as predictors of optimal learning experience among high-achieving Bedouin students. *Social Psychology of Education, 21*(1), 223–255.
- Kiemer, K., Gröschner, A., Pehmer, A.-K., & Seidel, T. (2015). Effects of a classroom discourse intervention on teachers' practice and students' motivation to learn mathematics and science. *Learning and Instruction, 35*, 94–103.
- Knopik, T., & Oszwa, U. (2019). Self-determination and development of emotional-social competences and the level of school achievements in 10–11-year-old Polish students. *Education 3-13, 48*(8), 972–987. DOI: 10.1080/03004279.2019.1686048
- Knopik, T., & Oszwa, U. (2022). Developing transferable competences of students: The self-determination theory and challenges of future education. *Lubelski Rocznik Pedagogiczny, 41*(1), 53–66. DOI: <http://dx.doi.org/10.17951/lrp.2022.41.1.53-6>
- Kunter, M., Tsai, Y.-M., Klusmann, U., Brunner, M., Krauss, S., & Baumert, J. (2008). Students' and mathematics teachers' perceptions of teacher enthusiasm and instruction. *Learning and Instruction, 18*, 468–482. DOI: /10.1016/j.learninstruc.2008.06.008
- Leroy, N., & Bressoux, P. (2016). Does amotivation matter more than motivation in predicting mathematics learning gains? A longitudinal study of sixth-grade students in France. *Contemporary Educational Psychology, 44–45*(2016), 41–53.

- Liu, W., Wang, C., & Ryan, R. (2016). *Building autonomous learners: Perspectives from research and practice using self-determination theory*. Springer.
- Lohbeck, A. (2018). Self-concept and self-determination theory: Math self-concept, motivation, and grades in elementary school children. *Early Child Development and Care*, 188(8), 1031–1044. DOI: 10.1080/03004430.2016.1241778
- Ma, X., & Xu, J. (2004). The causal ordering of mathematics anxiety and mathematics achievement: A longitudinal panel analysis. *Journal of Adolescence*, 27(2), 165–179.
- Magne, M., & Deci, E. (2005). Self-determination theory and work motivation. *Journal of Organizational Behavior*, 26(4), 331–362.
- Niemiec, C., & Ryan, R. (2009). Autonomy, competence and relatedness in the classroom: Applying self-determination theory to educational practice. *Theory and Research in Education*, 7(2), 133–144, DOI: 10.1177/1477878509104318
- Noels, K., Pelletier, L., Clement, R., & Vallerand, R. (2000). Why are you learning a second language? Motivational orientations and self-determination theory. *Language Learning*, 50(1), 57–85.
- Oga-Baldwin, W., Nakata, Y., Parker, P., & Ryan, R. (2017). Motivating young language learners: A longitudinal model of self-determined motivation in elementary school foreign language classes. *Contemporary Educational Psychology*, 49(2017), 140–150.
- Orsini, C., Binnie, V., & Tricio, J. (2018). Motivational profiles and their relationships with basic psychological needs, academic performance, study strategies, self-esteem, and vitality in dental students in Chile. *Journal of Educational Evaluation for Health Professions*, 15(11). DOI: 10.3352/jeehp.2018.15.11
- Osgood, C., Suci, D., & Tannenbaum, P. (1957). *The measurement of meaning*. University of Illinois Press.
- Papanastasiou, C. (2008). A residual analysis of effective schools and effective teaching in mathematics. *Studies in Educational Evaluation*, 34(1), 24–30.
- Pellegrino, J. (2012). *Education for life and work: Guide for practitioners*. National Research Council of the National Academies.
- Reeve, J. (2002). Self-determination theory applied to educational settings. In E. Deci & R. M. Ryan (Eds.), *Handbook of self-determination research* (pp. 183–203). University of Rochester Press.
- Sawin, G. (2004). General semantics for better transferable job skills. *A Review of General Semantics*, 61(1), 147–150.

- Shah, S. (2013). The use of group activities in developing personal transferable skills. *Innovations in Education and Teaching International*, 50(3), 297–307.
- Sternberg, R. (2003). *Wisdom, intelligence, and creativity synthesized*. Cambridge University Press.
- Stevens, S., & Miretzky, D. (2014). The foundations of teaching for diversity: What teachers tell us about transferable skills. *Multicultural Education*, 3, 30–40.
- Sutter-Brandenberger, C., Hagenauer, G., & Hascher, T. (2018). Students' self-determined motivation and negative emotions in mathematics in lower secondary education: Investigating reciprocal relations. *Contemporary Educational Psychology*, 55, 166–175.
- Trenshaw, K., Revelo, R., Earl, K., & Herman, G. (2016). Using self-determination theory principles to promote engineering students' intrinsic motivation to learn. *International Journal of Engineering Education*, 32(3), 1194–1207.
- Vallerand, R. (2001). Deci and Ryan's self-determination theory: A view from the hierarchical model of intrinsic and extrinsic motivation. Commentaries. *Psychological Inquiry*, 11(4), 312–318.
- Van Roekel, D. (2001). *Preparing 21st-century students for a global society: An educator's guide to the "Four Cs"*. National Education Association.
- Vansteenkiste, M., Lens, W., & Deci, E. (2006). Intrinsic versus extrinsic goal contents in self-determination theory: Another look at the quality of academic motivation. *Educational Psychologist*, 41(1), 19–31. DOI: 10.1207/s15326985ep4101_4
- Young, M., Depalma, A., & Garrett, S. (2002). Situations, interaction, process and affordances: An ecological psychology perspective. *Instructional Science*, 30(1), 47–63.