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Mathematical Resilience as a Conceptual Framework for School Practice

Abstract

The aim of the study is to analyze mathematical resilience from theoretical and practical perspectives. The paper is based on a review of the literature. Research on mathematics education has revealed a large proportion of students with math anxiety whose achievements in math are worsening. Such results encourage a discussion on preventing math anxiety. Research on the resilience of children and adolescents has accelerated in recent decades, resulting in the formulation of the concepts of school and mathematical resilience. The article presents the ecological model of mathematical resilience. The role of parents and teachers in promoting mathematical resilience is also described. The paper presents areas where students can self-foster mathematical resilience. The conclusions are related to the importance of strengthening mathematical resilience in education.

Keywords: mathematical resilience, math education, growth mindset, self-determination, self-efficacy

Introduction

As a result of research on math anxiety in the fields of education and psychology, studies on the phenomenon of school and mathematical resilience have been increasingly common in the international arena (Johnston-Wilder & Lee, 2010; Lee & Johnston-Wilder, 2017). The term resilience in psychology has been used to describe a set of abilities that allow a person to avoid the consequences of a difficult situation or even achieve success and experience personal growth despite failures. In the social sciences, this phenomenon was first described in the 1970s (Luthar et al., 2000; Masten, 2001). With regard to mathematics education, the term used in the past was the student's *emotional hardiness* in mathematics (Ariyanto et al., 2017), understood as permanent features and adaptive abilities that allow an individual to develop positively despite struggles and stress.

In the context of immune resources, resilience refers to the dynamic process of the interplay of biological, psychological, social, and environmental factors which constantly interact. They enable the individual to regain mental well-being despite experiencing adversities, even very challenging ones (Luthar & Brown, 2007; O'Leary, 1998). It is visible when experiencing serious difficulties while striving for development, despite objectively unfavorable circumstances (Oszwa et al., 2017).

In research on the resilience of children and adolescents, the following main trends emerge: a) identification of developmental risk and protective factors of resilience, b) analysis of the dynamics and mechanisms of resilience, c) strategies of strengthening resilience in people at risk, and d) multi-directional interdisciplinary research on resilience with strong input from neuroscience (see Sikorska, 2016).

Mathematical Resilience in Math Education

In the field of education, Clare Lee and Sue Johnston-Wilder (2017) suggested that mathematical resilience should become the term used to describe a student maintaining positive emotions toward mathematics

despite difficulties that they may experience and associate with the subject. Such an approach, which assigns a specific attribute of resilience, associating it with a given field (for example, mathematical resilience or sports resilience) is consistent with the partial approach to wellbeing. On the one hand, wellbeing can be discussed as a general assessment of one's own life, but on the other hand, it could be perceived as physical, spiritual, or creative well-being. Thus, mathematical resilience is a special type of resilience, an emotional and mental phenomenon related to the ability to return to a state of equilibrium despite periodic difficulties in learning mathematics.

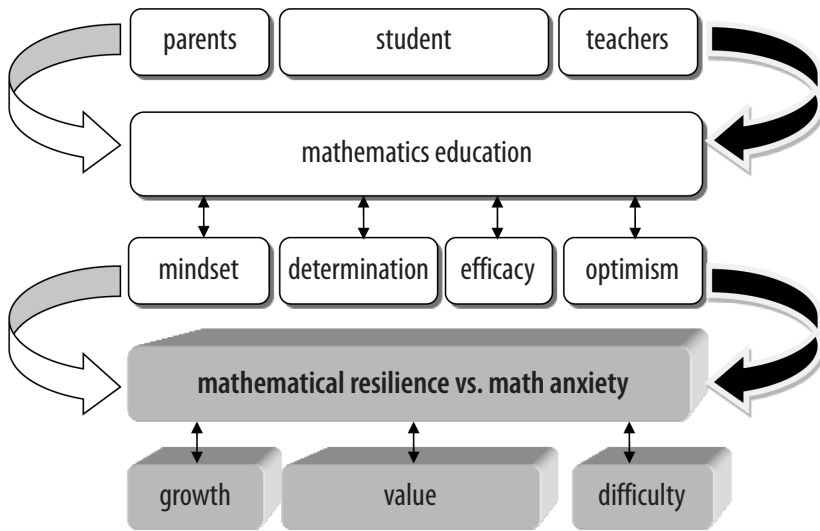
Focusing on the positive aspects of mathematics education can prove to be beneficial, not only in the context of preventing math anxiety, but also in making mathematics a field of knowledge accessible to many, and not reserved only for the mathematically gifted students. This is especially important in view of the research on gender stereotypes in mathematics (Bieg et al., 2015) and the threat that this stereotype poses among girls (Finnigan & Corker, 2016).

Analyses of students who manifest resilience in life situations consistently reveal the presence of three factors: a) individual immune resources, b) emotional support from parents, and c) a supportive psychosocial environment that encourages interaction, a creative approach to tasks, cooperation, and the development of problem-solving skills (Johnston-Wilder & Lee, 2010). Researchers have pointed out that students' mathematical resilience can be developed by emphasizing the value of effort in the education process and encouraging the student to make further attempts despite failures (Pieronkiewicz & Szczygieł, 2019). It is important for the teacher to have a flexible approach to the student, focusing on their development and encouraging them to take up challenges with attention on effort being the driving force of success (Dweck, 2017).

Ecological Model of Mathematical Resilience

In the comprehensive model of mathematical resilience designed by Clare Lee and Sue Johnston-Wilder (2017), the authors take into account its psychosocial background, as well as the consequences of acting in the process of learning/teaching mathematics. Its ecological dimension is presented in Figure 1.

Figure 1. Ecological model of mathematics education focused on the development of math resilience rather than math anxiety



Note. For the full description of the concept, see Johnston-Wilder & Lee (2010); Lee & Johnston-Wilder (2017).

There is no doubt that not only the students, but also their teachers and parents, participate in the process of mathematics education. The mathematics education system is multi-track, also in terms of the content being taught. It is accompanied by the construction of knowledge, while the acquisition of soft skills and competences – such as resilience, perseverance, and determination – are in high demand in this process.

Resilience, as the ability to activate resources of growth and development in problematic situations, may turn out to be a useful, universal life skill for dealing with difficulty and adversity; therefore, it could be useful in solving all types of problems, not only mathematical ones (Dweck, 2017). In this process, the student's mindset could be considered one of the keys of the math resilience model. Another key factor in math resilience is the student's motivation, especially their self-determination in forming intrinsic motivation (Deci & Ryan, 1985; Ryan & Deci, 2000a; 2000b). In developing math resilience, the emphasis has also been on perseverance and steadfastness, not giving up despite difficulties, and not breaking under the pressure of adversity. It is an opportunity to strengthen the sense of self-efficacy once a goal has been achieved (Bandura, 1982; 2000; 2006). Thus, another key element in the ecological model of math resilience, indicated by Lee and Johnston-Wilder (2017) is developing students' optimistic self-confidence in mathematics. This could be accomplished by encouraging them to persistently seek solutions to mathematical problems, to approach problems from different perspectives, and to exceed their own limitations. In line with Martin Seligman's (1995) concept of pedagogical optimism, a child with an optimistic explanatory style perceives obstacles as temporary and related to the situation, not as arising from their immutable personality traits.

Growth Mindset and Self-Determination in Math Resilience

Carol Dweck (2017) distinguished two types of mindset. The term is used to describe how an individual perceives the causes of their successes and failures. It could be focused on growth and development (growth mindset) or on the student's innate abilities (fixed mindset). A person with a growth mindset believes that intelligence, special abilities, and skills can be developed and that they can be improved through effort, which gives unlimited development opportunities in any field. The research by Dweck (2017) looked at the overall learning process. In adapting Dweck's theory to mathematics education, it seems that activities aimed at changing

beliefs and focusing the mentality on student growth and development should be long-term and on a larger scale. Some researchers (e.g., Ramirez et al., 2018) suggest designing studies that indicate strategies for systemic influences that can change students' mindsets toward more constructive interpretations of mistakes and failures in mathematics learning.

Another element of the dynamic model of math resilience is self-determination. It originates from self-determination theory, which assumes that it is inherent in human nature to be curious about one's environment and interested in learning and development (Deci & Ryan, 1985; Ryan & Deci, 2000b). Satisfying children's basic psychological needs for autonomy, competence, and relatedness are crucial to their personal development. In education, it facilitates students' autonomous self-regulation of learning and self-satisfaction. The most autonomous type of motivation is intrinsic motivation, which refers to behaviors that are interesting and enjoyable to the individual (Ryan & Deci, 2000a). Intrinsically motivated behavior is sustained by an internal sense of interest and satisfaction. On the contrary, external motivation results from incentives beyond the behavior itself, such as rewards and external reinforcement. A self-determined student will be characterized by more autonomous motivation, consistent with their intrinsic goals. This type of motivation is opposite to controlled motivation, which is related to extrinsic goals (Ryan & Deci, 2017). In the math resilience model, the key would be also to meet the universal needs of the student in terms of developing autonomous motivation. As indicated in a recent study (Knopik & Oszwa, 2019), constructing educational situations aimed at meeting the needs of students' relatedness, competence, and autonomy creates ample opportunity to take responsibility for the learning process.

Both the self-determination theory (Deci & Ryan, 1985), and the theory of mindsets (Dweck, 2017) seem to gain special recommendation in mathematics education, ensuring an increase in students' intrinsic motivation and an attitude of making more of an effort along with faith that difficulties can be overcome. Against this background, it is particularly important for parents and teachers to develop such an attitude in students, which would be an investment not only in their success

in mathematics, but also in many other areas of life and activity (Johnston-Wilder & Lee, 2010).

Self-Efficacy and Optimism in Math Resilience

For success in mathematics, it is important to develop students' confidence in the subject. One way to do so is to encourage students to persistently look for solutions and approach problems from multiple perspectives and to encourage them to exceed the limits of their skills. Math education for the development of pedagogical optimism (Seligman, 1995) in the process of learning and teaching is a particularly important element in acquiring mathematical knowledge. The difficulty lies in the inherent nature of math problems. This means that students who do not give up too easily in the process of overcoming difficulties will also be less prone to experience math anxiety. Thus, developing pedagogical optimism has a clear correlation to reducing the level of negative emotions in learning math. Moreover, perseverance recruits support when it is needed (Beilock & Willingham, 2014; Oszwa et al., 2017). Researchers point to differences between persistence and perseverance, although both are important for resilience. Perseverance is related to knowing when to continue without giving up too soon, how to apply alternative strategies, and when to ask for support (Lee & Johnston-Wilder, 2017).

Mathematics education can be seen as opportunity to develop students' self-efficacy, both in a general sense and specifically in relation to mathematical activities. The concept of self-efficacy was introduced into psychology by Albert Bandura (2000) and it refers to the subjective feeling about our expectation of how well we will cope with a task. Expectations may include a) self-efficacy (Can I do this task?), b) performance (How well will I do this task?), and c) utility (What benefit will completing this task bring?). The stronger the students' sense of self-efficacy, the greater their perseverance and determination will be in pursuing their goals. Self-efficacy affects the degree of commitment to action. The more

the students are convinced of their ability to achieve a goal, the more effort they will be able to put into achieving it (Bandura, 2006).

The ecological model of math resilience is a theoretical construct that will need empirical verification, which is usually a multi-stage process. It requires the construction of tools to measure its components as well as to determine their role in the model. Teaching practice can also provide some data to expand the theoretical concept of the model.

Math Resilience in School Practice

Both parents and teachers have a modelling influence on the attitudes connected with ability and effort in learning mathematics. Parents, as the first educators of their children, have many opportunities to interest them in mathematics, to show its many dimensions, and to encourage persistence in overcoming difficulties. Early education teachers can communicate positive attitudes toward mathematics and its value and can arouse curiosity and interest in numbers and mathematics in the surrounding world. Teachers also play a role in the early identification of students at risk of mathematics anxiety, which may prove useful in understanding its determinants and mechanisms. Knowing how to promote and foster math resilience at school is important for teachers and parents, as well as for the students themselves. The repertoire of such strategies may be different for each group. Math resilience is associated with placing value on mathematics, having a growth mindset, and the ability to recognize that math problem-solving usually involves effort and challenges. This is why the availability of support is so important in math resilience. The specific support mechanism has been presented in the growth zone model (Lee & Johnston-Wilder, 2017), based on Vygotsky's concept of the zone of proximal development.

The Role of the Teacher in Fostering Students' Math Resilience

Students experiencing failure in mathematics may be one of the potential causes of math anxiety. The teacher plays a role in fostering both math resilience and math anxiety. Math anxiety can arise because of math itself, as knowledge becomes complicated in a spiral way and tasks become more difficult. However, it has often been a consequence of the way the subject was presented and taught by the teacher. Moreover, math anxiety may be transmitted by teachers through their negative experiences from their time as students (Beilock et al., 2010). This transfer seems particularly strong from pre-service female teachers to their female students (Szczygieł & Cipora, 2016). Furthermore, this modelling influence may be reinforced by teachers' false belief that mathematics is the domain of men (Turska & Oszwa, 2017; 2018). Raising awareness among math teachers about the math gender stereotype and its impact on girls' mathematical achievements may require long-term efforts to change attitudes and the methods of communication and assessment.

In order to promote math resilience, math teachers should give students the opportunity to discover new rules and formulas and should encourage them to struggle. According to the psychological principle, the most constructive thinking occurs when we struggle with a problem of moderate difficulty (Dowker et al., 2016).

As evidenced by students' reports, teachers often send them verbal messages that intensify their math-related frustrations instead of reducing them. According to Dweck (2017), a message that emphasizes the student's effort and communicates the teacher's belief in their potential has the greatest motivating value. Stimulating a growth mindset in students from the early stages of education is far more motivating than developing a fixed mindset, which in the future may manifest as explaining one's lack of success in mathematics with a lack of mathematical skills ("I don't have math skills; therefore I can't do it"). The teacher's belief in their students' abilities and appreciation of their commitment to solving problems are more likely to provide a positive experience and to develop math resilience rather than math anxiety.

Modifications in teacher education also include a promising direction in promoting math resilience. Both math teachers and early education teachers should participate in programs which enhance their knowledge of math resilience and strategies that can foster it (Beilock & Willingham, 2014). Their contact with children in early education has an effect on the long-term attitudes and emotions toward school subjects and difficulties (Beilock, 2008).

Parents' Role in Promoting Their Children's Math Resilience

When parents communicate positive attitudes toward mathematics, paying attention to personal statements about mathematics as a school subject and their experiences with it in the presence of children, they can promote math resilience by avoiding negative modelling ("I never knew math, math was always difficult for me"). Studies on parents helping children with their math homework (Herts et al., 2019; Vukovic et al., 2013) indicate the controversial nature of this practice. On the one hand, support from parents and adequate expectations toward the child may be strong factors in developing math resilience and it may have a positive impact on their achievement in math (Vukovic et al., 2013). On the other hand, mothers who are anxious about mathematics may aggravate this anxiety in their children, so it seems better if they do not engage in their children's math homework (Maloney et al., 2015) until they have addressed their own anxiety.

Students as the Authors of Their Own Math Resilience

The student is an active subject of mathematics education, and them correctly interpreting situations and giving meaning to experiences with mathematics can be considered a key factor for the development of math resilience. In forming math resilience, it is important to understand that difficulties, barriers, and obstacles are an integral part of the learning

process, which can give the struggle in mathematics a new meaning (Lee & Johnston-Wilder, 2017). An attempt to reformulate the assessment of a mathematical situation and one's own reactions to it may help one become convinced that learning can be useful even when it is not carried out with ease (Dweck, 2017; Johnston-Wilder & Lee, 2010; Pieronkiewicz & Szczygieł, 2019).

It may also be of particular importance to assign meaning to signals of physiological arousal. Most people see them as a general response to a stressful situation, a threat that should be avoided. However, some people assess such physiological indicators as stimulation, the announcement of a challenge that needs to be faced and overcome. Such interpretative differentiation leads to different strategies for coping with stress (Ramirez et al., 2018). Physiological markers of anxiety may be assessed by some as a threat and by others as a challenge (Oszwa, 2020). The arguments for the effectiveness of such an approach have been provided by Jameson (2014) in a study on two groups of students at higher vocational schools: 1) an experimental group, in which students with math anxiety were informed before a test that increased emotional arousal is optimal for solving mathematical tasks, and 2) a control group, in which students were advised to ignore any signs of stress when taking a test and to carry on regardless. After the test, significantly more correct answers were observed in the experimental group than in the control group (Jameson, 2014). This indicates that an awareness of emotional arousal and a positive assessment of its symptoms may have an impact on students' math performance.

Research (Lin-Siegler et al., 2016) has positively verified the assumption that students who hear about the difficulties of famous scientists may feel more connected to the field of knowledge they are learning (Bedyńska & Dreszer, 2006). A particularly noticeable improvement in performance was seen in students with lower achievement. In science – including math – failure is especially important, as it carries a lot of information and allows one to learn from errors. Therefore, the realistic images of scientists who made errors yet achieved a lot may encourage perseverance, even in spite of obstacles (Lin-Siegler et al., 2016).

Conclusion

Math resilience seems to be a promising construct that can lead to good practice in mathematics education. In the contemporary world, there has been noticeable increase in importance of mathematics as a field of knowledge. On the other hand, a rise in difficulties studying mathematics has also been reported in many countries. This situation has urged researchers to explore the field multidirectionally. One trend is to analyze the causes and mechanisms of math difficulties. Another direction of research is based on a discussion of what differentiates mathematics from other school subjects, leading to the discovery and empirical exploration of the math anxiety that accompanies math education and is related to lower math achievement. In this context, the interest in math resilience seems to lead to the prevention of math anxiety, as well as higher achievements in mathematics. The road may turn out to be long and fraught with challenges, but it brings hope for the improvement in the educational situation of many students. Math resilience being represented in the pedagogical literature is an important step in forming the concept, planning related research, constructing models, doing empirical research, and eventually implementing the results into educational practice for individual and social benefits.

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